Monte Carlo simulations of the microwave emissivity of the sea surface

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Abstract. A Monte Carlo model is developed to calculate the microwave emissivity of the sea surface based on the Kirchhoff approximation combined with modified Fresnel coefficients. The modified Fresnel coefficient depends on the incident angle of the electromagnetic wave and the height variance of small-scale roughness, which is an approximation to account partly for the scattering effect from small ripples. The advantage of the Monte Carlo model is its inherent capability to treat multiple scattering events. Using a two-dimensional Gaussian distribution for the sea surface slope variability, the model is capable of simulating the azimuthal dependency of the microwave emission caused by the alignment of waves perpendicular to the wind direction. Good agreement between model calculations and measurements is obtained.

1. Introduction

Accurate knowledge of reflection and emission characteristics of the ocean surface as a function of surface conditions is a prerequisite for deriving microwave retrieval algorithms [e.g., Bauer and Schluessel, 1993; Karstens et al., 1994; Kummerow and Giglio, 1994] for surface and atmospheric parameters over oceanic areas. The reflection and emission characteristics of the ocean surface depend on the dielectric constant, which is a function of temperature and salinity, and on the roughness of the ocean surface. The latter is usually divided into the following three parts: roughness elements larger than the electromagnetic wavelength described by a surface element normal distribution (the so-called stationary phase approximation), subscale surface roughness, and foam [e.g., Guissard et al., 1986; Trokhimovski and Irisov, 1995; Wisler and Hollinger, 1977; Wenz, 1975; Wu and Fung, 1972]. These parameters are usually estimated as functions of meteorological conditions (e.g., wind speed, stability, and temperature).

The two-scale (small irregularities superimposed on large undulations) scattering model has been well described by Wu and Fung [1972] and Wenz [1975]. The range of validity of the model for the perfect conducting random surface is discussed by Pan and Fung [1987].

Among the motivations for our study are to overcome difficulties that arise for large incidence angles [Guissard et al., 1994] and to consider multiple scattering events by using the Monte Carlo approach. Another goal is to study three-dimensional radiative effects from clouds, together with the rough surface. In section 2 we discuss the characteristics of the sea surface in the field of remote sensing. In section 3 we describe our Monte Carlo method in detail. In section 4, comparisons are carried out between model calculations and measurements. In section 5, first results are discussed.

2. Foam and Roughness of Sea Surface

Foam is produced by the mixture of air and water and is generated by large wind speeds. The air volume fraction in the foam is about 0.95. The foam coverage may be expressed by [Tang, 1974]

\[ d = 7.75 \times 10^{-6} \left( \frac{V}{V_0} \right)^{3.231} \]  

(1)

where \( V \) is the wind speed in meters per second at 10 m above sea surface and \( V_0 \) is a constant of 1 m s\(^{-1}\). The total reflectivity is calculated from the sum of the foam reflectivity weighted with the foam coverage \( d \) and the reflectivity of water weighted with the water coverage \( 1 - d \).

The sea surface roughness is commonly described by the roughness spectrum of the sea surface. We choose the angular-independent roughness spectrum \( S(K) \) from Bjerkaas and Riedel [1979] and an angular function of \( \cos 2 \alpha \) according to Pierson et al. [1955], where \( K \) is the wavenumber and \( \alpha \) is the azimuth angle away from the upwind direction. The roughness spectrum \( S(K) \) is assumed to be a function of the friction velocity of the wind only. The mean slope variance of a rough sea surface is then calculated from its roughness spectrum by

\[ \sigma^2 = \frac{1}{\pi} \int_{-\infty}^{\infty} K^2 S(K) dK \cos^2 \alpha d\alpha \]  

(2a)

The slope variance along the upwind and downwind direction is then given by
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The low limit of the friction velocity in the surface roughness spectrum that we used here is 12 cm s\(^{-1}\). The low limit is acceptable for the application when considering the history of waves and air-sea interactions. The friction velocity of 80 cm s\(^{-1}\) is a reasonable upper limit for (4).

Figure 1 shows that \(K_c\) increases with increasing frequency \(f\) of the electromagnetic wave and friction velocity of the wind. The behavior can also be seen from (4). For a constant friction velocity the roughness spectrum \(S(K)\) is fixed. The left side of (4) decreases with an increase of the wavenumber \(k\) (equivalent to frequency \(f\)), which requires an increase of \(K_c\) to maintain (4). For a given \(f\) of the electromagnetic wave, the right side of (4) increases with friction velocity because the roughness spectrum \(S(K)\) at high frequencies increases rapidly with the increase of the friction velocity, so that it requires an increase of \(K_c\) to remain the equal sign of (4). Although (4) holds, (3a)-(3c) will fail for very high wind speeds. Similar to (2a)-(2c), the surface slope variance for the large-scale surface roughness is calculated from

\[
\tilde{\xi}^2 = \frac{1}{\pi} \int_0^\infty \int_{-\pi}^\pi K^2 S(K) \, dK \cos^2 \alpha \, \cos \alpha \, d\alpha = \frac{3}{4} \sigma^2 \quad (5a)
\]

The slope variance along the upwind and downwind direction is given by

\[
m_u^2 = \frac{1}{\pi} \int_{-\pi}^\pi K^2 S(K) \, dK \cos^2 \alpha \, \cos \alpha \, d\alpha = \frac{3}{4} \tilde{\xi}^2 \quad (5b)
\]

and the slope variance along the crosswind direction is given by

\[
m_v^2 = \frac{1}{\pi} \int_{-\pi}^\pi K^2 S(K) \, dK \sin^2 \alpha \, \cos \alpha \, d\alpha = \frac{1}{2} \tilde{\xi}^2 \quad (5c)
\]

3. Methodology

We use the Monte Carlo method to simulate stochastic processes of photons [Liu et al., 1996] for single and multiple reflection events at the sea surface. As discussed in section 2, the method is to combine the limit of Kirchhoff approximation with part of the scattering effect from small-scale roughness separated by the cutoff wavenumber \(K_c\). The small-scale surface roughness is superimposed onto the large-scale surface roughness. The emissivity of the sea surface at an observation zenith angle \(\theta\) is defined by

\[
\begin{bmatrix}
\varepsilon_v(\theta) \\
\varepsilon_h(\theta)
\end{bmatrix} = \begin{bmatrix}
1 \\
1
\end{bmatrix} - \begin{bmatrix}
\Gamma_v(\theta) \\
\Gamma_h(\theta)
\end{bmatrix}
\]

where \(\Gamma\) is reflectivity. The subscripts \(v\) and \(h\) denote the vertical and horizontal polarization, respectively. In this paper we assume that \(\theta > 90^\circ\) is for the downward direction (from the atmosphere to the ocean). For a calm water surface the reflectivity can be described by Fresnel's law, i.e., a specular reflection model. For large-scale surface roughness the scattering is treated according to the Kirchhoff model under the stationary-phase approximation. In this approximation the large-scale surface roughness is assumed to be a set of tangent planes or facets, in which the local reflection obeys Fresnel's law. The total reflectivity is then obtained by averaging the Fresnel reflection coefficients of the individual facets weighted with the slope probability density distribution. The commonly used Gaussian
distribution for the surface slope is adopted here. The reflectivity for the large-scale surface roughness is then written as [e.g., Ulaby et al., 1981; Tsang et al., 1985]

\[ 
\Gamma_s(\theta) = \frac{1}{4\pi|\cos \theta|} \int_0^{2\pi} \int_0^{\pi/2} S \left( \begin{array}{c} 1 \\ 1 \\ q^4 \\ \frac{1}{2m_\mu m_\sigma} \end{array} \right) \left( \begin{array}{c} q_x^2 + q_y^2 \\ q_x^2 + q_z^2 \end{array} \right) \sin \theta, d\theta, d\phi, 
\]

where

\[ q_x = \sin \theta \cos \phi - \sin \theta \cos \phi \]
\[ q_y = \sin \theta \sin \phi - \sin \theta \sin \phi \]
\[ q_z = \cos \theta - \cos \theta \]
\[ q^2 = q_x^2 + q_y^2 + q_z^2 \]

with the definitions

\[ a = \left( \sin \theta \cos \theta - \sin \theta \cos \phi \right)^2 \]
\[ b = \left( \sin \theta \sin \phi - \sin \theta \sin \phi \right)^2 \]
\[ \cos 2\theta_i = -\cos \theta \cos \theta_i - \sin \theta \sin \theta_i \cos (\phi_i - \phi) \]

in which \( \epsilon_w \) is the complex relative dielectric constant of water.

The local Fresnel reflection coefficients for the local incident angle \( \theta_i \) are

\[ |R_x(\theta_i)|^2 = \left| \frac{\cos \theta_i - (\epsilon_w - \sin^2 \theta_i)^{1/2}}{\cos \theta_i + (\epsilon_w - \sin^2 \theta_i)^{1/2}} \right|^2 \]
\[ |R_y(\theta_i)|^2 = \left| \frac{\epsilon_w \cos \theta_i - (\epsilon_w - \sin^2 \theta_i)^{1/2}}{\epsilon_w \cos \theta_i + (\epsilon_w - \sin^2 \theta_i)^{1/2}} \right|^2 \]

Figure 2. Differences between the original and modified Fresnel reflection coefficient for a friction velocity of 50 cm s\(^{-1}\) and a sea surface temperature of 291 K for vertical (solid line) and horizontal (dashed line) polarization. The viewing direction is from nadir.

Therefore two random numbers \( R_1 \) and \( R_2 \) describe the statistics of the tilting angles \( \theta_n \) and \( \phi_n \). Once \( \theta_n \) and then \( \phi_n \) are determined from \( R_1 \) and \( R_2 \), the local incident angle \( \theta_i \) and the scattering direction can be calculated as follows:

\[ \cos \theta_i = -\cos \theta \cos \theta_n + \sin \theta \sin \theta_n \cos (\phi_i - \phi_n) \]
\[ \sin \theta_i \cos \phi_n = \sin \theta \cos \phi + \cos \theta \sin 2\theta_n \cos \phi_n \]
\[ -2 \sin \theta \sin^2 \theta_n \cos \phi_n \cos (\phi_i - \phi_n) \]
\[ \sin \theta_i \sin \phi_n = \sin \theta \sin \phi + \cos \theta \cos 2\theta_n \sin \phi_n \]
\[ -2 \sin \theta \sin^2 \theta_n \sin \phi_n \cos (\phi_i - \phi_n) \]
\[ \cos \theta_i = \sin \theta \sin 2\theta_n \cos (\phi_i - \phi_n) - \cos \theta \cos 2\theta_n \]

The reflected part of the photon is then written as

\[ \Gamma_s(\theta) = \frac{1}{4\pi|\cos \theta|} \int_0^{2\pi} \int_0^{\pi/2} S \left( \begin{array}{c} 1 \\ \frac{m_\mu m_\sigma \cos \theta_i}{m_\mu^2 \cos^2 \theta_i + m_\sigma^2 \sin^2 \phi_n} \cos \theta_n d\theta_n d\phi_n \right) \]

With the definitions

\[ dR_1 = \frac{d\phi_n}{2\pi} \]
\[ dR_2 = \left( \frac{\cos^2 \phi_n + \sin^2 \phi_n}{m_\mu^2 + m_\sigma^2} \right) \]
\[ \exp \left[ \frac{-\tan^2 \theta_n}{2} \left( \frac{\cos^2 \phi_n + \sin^2 \phi_n}{m_\mu^2 + m_\sigma^2} \right) \right] \sin \theta_n d\theta_n \]

we can write

\[ \gamma_s = \frac{c}{|\cos \theta|} S \left( \begin{array}{c} 1 \\ \frac{m_\mu m_\sigma \cos \theta_i}{m_\mu^2 \sin^2 \phi_n + m_\sigma^2 \cos^2 \phi_n} \cos \theta_n \right) \]

If \( \theta_n > 90^\circ \), the photon is assumed to undertake another run through the Monte Carlo simulation.
reflection event (i.e., multiple scattering events) with the water surface. The reflection part for the photon after \( k \) reflection events can be expressed as

\[
\gamma_r = \frac{1}{\cos \theta} \cdot c \cdot S_1 \cdot S_2 \cdot \cdots \cdot S_k \cdot S_1
\]

(16)

Where \( c \) (see (15h)) and \( S_k \) (see (9)) depend on the incoming and outgoing directions of the \( k \)th reflection event. Upward directed photons (i.e., \( \theta < 90^\circ \)) can also take another reflection event with the surface, but the possibility of this should be small. This case is not considered in the present model owing to the limit imposed by the representation of the ocean surface elements by a surface normal distribution. The same circumstances prohibit the exact inclusion of shadowing. Another assumption is that the distribution of \( S(k) \) remains the same for all scattering processes, which is not exactly true.

The electromagnetic wave can be reflected and diffracted by small ripples before and after the reflection from the background large-scale facets. Comprehensive treatments of the scattering effects from small ripples can be found in the literature [e.g., Wu and Fung, 1972; Wentz, 1975]. We use a simple approximation (modified Fresnel coefficient) to account partly for the scattering effects. We follow Guissard and Sobieski [1987] by using

\[
|R_v|^2 = |R_v|^2 \exp (-4k^2 \zeta R \cos^2 \theta)
\]

(17a)

\[
|R_h|^2 = |R_h|^2 \exp (-4k^2 \zeta H \cos^2 \theta)
\]

(17b)

Differences between the original and the modified Fresnel reflection coefficients reach a maximum at about 20 GHz, then decrease with frequency (Figure 2). Both \( k \) and \( \zeta \) are the function of frequency. For large wind speeds, foam has to be taken into account. The complex relative dielectric constant of foam can be parametrized according to Droppleman [1970]

\[
e_f = e_w \left[ 1 - \frac{3V_a(e_w - 1)}{(2e_w + 1) + V_a(e_w - 1)} \right]
\]

(18)

where \( V_a \) is the air volume fraction in foam, assumed to be between 0.94 and 0.99. The reflectivity of foam depends also...
on the thickness of the foam layer [e.g., Schrader and Liu, 1995]. Stogryn [1972] derived an analytical expression for foam emissivity from measurements that was between 13.4 and 37 GHz. In this paper the reflectivity of the foam is calculated as a function of sea surface temperature, frequency, and surface wind speed, following Ulaby et al. [1986].

The microwave reflectivity of the sea surface can now be calculated from the Monte Carlo model by sending many photons (e.g., 10,000) from the detector direction \((\theta, \phi)\) to the sea surface. Using (11a), (11b), (15a)–(15h), and (16) we can calculate the reflection part for each photon. The reflectivity of the sea surface is then carried out by averaging the reflected parts of the photons.

4. Comparisons With Measurements

In order to test our algorithm, we performed two calculations, first with measurements at the North Sea and second with data given by Hollinger [1970]. Simultaneous measurements from the special sensor microwave/imager (SSM/I) and radiosonde and ship synoptic observations for clear sky cases were collected during the International Cirrus Experiment in October 1989. The ship measurements were performed from the German research vessel POSEIDON cruising on the North Sea. The time differences between satellite measurements and radiosonde and ship measurements were less than 15 min. The observed atmospheric parameters were used as input for our radiative transfer code [Liu and Ruprecht, 1996]. The calculated brightness temperatures agree, in general, with the satellite measurements (Figure 3). The rms error between the modeled and measured microwave brightness temperature \(T_b\) is less than 2 K for 19.35, 22.235, and 37 GHz. The rms error reaches 4 K for the horizontally polarized \(T_b\) at 85.5 GHz. We do not compare the vertically polarized component of the brightness temperature \(T_b\) at 85.5 GHz because this channel was not functioning at that time. Another comparison was performed with the surface measurements taken from Hollinger [1970]. The surface measurements are at 8.36 and 19.35 GHz for a sea surface temperature of 291 K and wind speeds of 0.5 and 13.5 m s\(^{-1}\) measured at 43 m above the sea surface. The errors of the measurements estimated by Hollinger [1970] are between 5% and 10%. Effects due to foam and the reflected atmospheric radiation were removed. The model calculations were carried out for the same frequencies and surface temperatures, but with wind speeds of 3.5 and 13.5 m s\(^{-1}\). A wind speed of 3.5 m s\(^{-1}\) instead of 0.5 m s\(^{-1}\) was applied in the calculations because 3.5 m s\(^{-1}\) corresponds to the minimum friction velocity of 12 cm s\(^{-1}\) in the sea surface roughness spectrum of Bjerkaas and Riedel [1979]. The model calculations were performed by setting \(m_u = m_c\) in (12) because the relative azimuth angle of the measurement of Hollinger [1970] is not known. For \(m_u = m_c\), a difference of a constant factor of 2 is found in the exponential expressions given by Ulaby et al. [1986], equation (18.33)] and Wu and Fung [1972, equation (32)]. Better agreement with Hollinger's [1970] measurements is achieved with the exponential expression by Wu and Fung [1972] (Figures 4a and 4b). The rms error between the modeled and measured \(T_b\) is 4.2 K for 8.36 GHz and 5.5 K for 19.35 GHz.

5. Sensitivity Calculations

Further calculations were conducted to show the applicability of the present method. The dependency of brightness temperature on azimuth angle of the viewing angle influences the retrieval of the sea surface wind from SSM/I measurements [Wentz, 1992]. We performed calculations to quantify the effect of azimuth angle variations on the upwelling \(T_u\) at the sea surface (Figure 5). The vertically polarized \(T_u\) reaches a maximum in the upwind-downward directions and a minimum in the crosswind direction. Opposite results are found for the horizontally polarized \(T_u\). These behaviors are similar to the measurements by Wentz [1992] and Yueh et al. [1995].
Multiple scatterings of photons at the sea surface can affect the total reflectivity. The percentage of photons that have undertaken two or more reflection events increases dramatically with the friction velocity of wind and the incidence angle of the electromagnetic wave (Figure 6). The percentage is less than 1% at nadir, but it increases to 20% at a viewing angle of 60° for a friction velocity of 150 cm s\(^{-1}\). The effects of single and multiple reflections on the microwave reflectivity of sea surface are shown in Figure 7, in which calculations were made at 19.35 GHz for a zenith angle of 60° and a sea surface temperature of 291 K. The total reflectivity is the sum from...
single and multiple reflections. Obviously, the contribution of multiple reflection to the vertically polarized component of the total reflectivity is small. However, the contribution of multiple reflections to the horizontally polarized component can reach almost 10% (Figure 7).

As mentioned in section 1, difficulties arise in numerical calculations from using (7) directly because the integrand is almost a delta function for large incidence angles and a near specular reflection direction. Such problems can be avoided by using the Monte Carlo method because we simulate individual photons instead of using the delta function explicitly. Model results for large angles seem reasonable, and negative emissivities do not appear.

6. Discussion

Although agreement between measurements and model calculations is achieved, some problems, such as the sea roughness spectrum and the criteria for the cutoff wavenumber, need to be studied further. Only part of the scattering effect from small ripples is considered with the modified Fresnel coefficient. The application of the present model is limited for calculating signatures of the passive microwave measurements such as SSM/I. The absolute error of the model calculation is estimated to be less than 3%. Since the model is a noncoherent one, it cannot correctly treat a perfectly conducting rough surface and anomalous microwave radiative temperatures over the ice surface, i.e., larger horizontally than vertically polarized microwave radiative temperatures. A more physical model [e.g., Wentz, 1975] is required to account for the diffractions of the electromagnetic wave from small ripples. Different choices for the roughness spectrum and the criteria for the cutoff wavenumber can slightly change the simulated surface emissivity. In addition, the microwave emissivity of the sea surface, which is calculated from the Gaussian slope distribution, is upwind-downwind symmetric. This symmetry conflicts with the upwind-downwind asymmetry in the scatterometer response [e.g., Sobieski et al., 1991]. However, the only way to incorporate these effects, to correctly include shadowing, and to completely describe multiple scattering is the replacement of the slope distribution model by a complete ocean surface topography.

References


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