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The continuous smooth hockey stick: a newly proposed spawner-recruitment model

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Summary

Spawner-recruit relationships are important components of fisheries management. The two most widely used models have been criticized for unsatisfactory fits and biologically unreasonable extrapolations. A simple hockey stick model has been shown to provide more robust predictions, however, this model is not widely used, possibly because the abrupt change from density-dependence to density-independence is unrealistic and the piecewise model is difficult to fit. Here I present a continuous two-parameter model that resembles a smoothed hockey stick and provides parameter estimates similar to the piecewise hockey stick. The new model is easily parameterized with regular curve-fitting routines.

Introduction

Understanding the relationship between the size of spawning stocks and the number of subsequent recruits remains one of the challenges in fisheries biology and management. The most widely used models fitted to respective data are the ones proposed by Beverton and Holt (1957) and Ricker (1954). However, these models have been criticized because they often lead to unsatisfactory fits and biologically unreasonable extrapolations (Myers et al., 1994; Myers et al., 1999; Barrowman and Myers, 2000). Barrowman and Myers (2000) suggest instead a piecewise linear spawner-recruitment model that fits a straight line through the origin to density-dependent recruitment and a second straight line parallel to the X-axis to density-independent recruitment. They analyse 246 data sets and show that this hockey stick model typically leads to more realistic fits and better predictions for the ability of populations to recover from low densities, and for the maximum carrying capacity of recruits. Despite these advantages the hockey stick model has not been widely adopted in fisheries management, probably because piecewise functions are difficult to fit and have cumbersome statistical properties. Barrowman and Myers (2000) also concede that the sharp bend of the hockey stick indicating an abrupt change from density-dependent to density-independent mortality is probably unrealistic and they therefore present a smoothed version, which, however, still consists of a piecewise function, with an additional parameter to determine smoothness. Here I present a continuous two-parameter model for a smooth hockey stick which can be easily parameterized with curve-fitting algorithms such as available in standard statistical software packages.

The new model

The new stock-recruitment model is shown in equation 1.

$$R = R_\infty\left(1 - e^{-\frac{\alpha}{R_\infty}S}\right) \tag{1}$$

where R is the abundance of recruits, S is the abundance of their spawners, R_∞ is the carrying capacity of recruits, and α is the slope at the origin of the spawner-recruitment curve. Assuming log-normal distribution of recruitment, a log transformed version of equation 1 must be fitted to log-transformed abundance data for recruits, as shown in equation 2.

$$\ln R = A + \ln\left(1 - e^{-\frac{\alpha}{e^A}S}\right) \tag{2}$$

where $A = \ln R_\infty$.

Figure 1 shows an application of the new model to spawner-recruitment data for Coho salmon (*Oncorhynchus kisutch*) at Needle Branch Creek, Oregon (Barrowman and Myers, 2000; Bradford et al., 2000). The data represent females migrating upstream to spawn (S), and the resulting female smolts migrating downstream approximately 1.5 years later (R). Hockey stick estimates for α are 39.5 (29.5–53.5) and for R_∞ 154.5 (Bradford et al., 2000). In comparison, the new function gives α as 53.4 (20.5–86.2) and R_∞ as 150.8 (113.1–201.1), i.e., similar estimates with overlapping 95% confidence limits.

Discussion and conclusion

The new stock-recruitment model retains the simplicity of the hockey stick by assuming density-dependence at low and density-independence at high spawner abundance. It overcomes the statistical and biological problems associated with

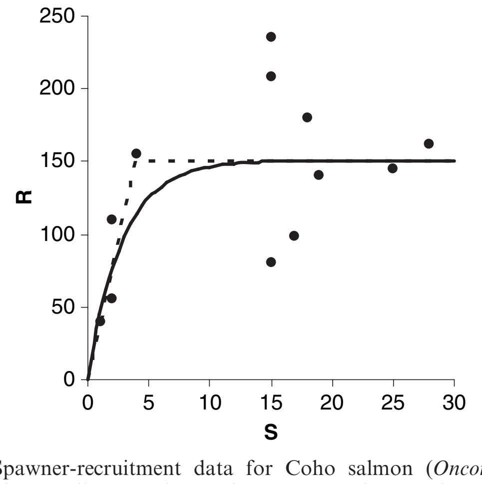

Fig. 1. Spawner-recruitment data for Coho salmon (*Oncorhynchus kisutch*) in Needle Branch Creek, Oregon. The X-axis represents number of females migrating upstream for spawning. The Y-axis represents number of female smolts migrating downstream 1.5 years later. The dotted line is the hockey stick model and the continuous line is the new S–R function

an abrupt change between these two states by providing a continuous two-parameter model with a smooth transition zone. Parameters can be easily estimated with the curve-fitting functions of widely available statistical software packages. It is hoped that this will lead to a wider application of the hockey stick approach. A larger study is underway which formally compares the results of the new stock-recruitment model with those obtained from the Ricker (1954) and Beverton and Holt (1957) models, with special emphasis on species with different reproductive strategies.

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