Quaas, Martin F.; Requate, Till

Working Paper
Sushi or fish fingers? Seafood diversity, collapsing fish stocks, and multi-species fishery management

Economics working paper / Christian-Albrechts-Universität Kiel, Department of Economics, No. 2012-03

Provided in cooperation with:
Christian-Albrechts-Universität Kiel (CAU)

sushi or fish fingers?
seafood diversity, collapsing fish stocks, and multi-species fishery management

by Martin F. Quaas and Till Requate
Sushi or Fish Fingers? Seafood Diversity, Collapsing Fish Stocks, and Multi-species Fishery Management

Martin F. Quaas§ and Till Requate¶
Department of Economics, University of Kiel, Germany
Kiel Institute for the World Economy, Kiel, Germany

October 2011

Abstract. We present a model of a multi-species fishery and show that (i) consumer preferences for seafood diversity may trigger a sequential collapse of fish stocks under open-access fishery, (ii) the stronger the preferences are for diversity the higher is the need for coordinated multi-species regulation, (iii) second-best optimal management of only one (or a few) species is less strict than socially optimal management of the same species. Finally, (iv) myopic regulation of one species, ignoring spill-overs to other species, may cause depletion of other stocks that would not be depleted under open access.

Keywords: marine biodiversity, fishery economics, product differentiation
JEL-codes: Q22, Q57, Q21
I Introduction

The overfishing problem has several dimensions. First, excessive exploitation of fish stocks in the short term leads to inefficiently low stocks in the long term. Secondly, more and more fish stocks are being fished to the point of collapse on a worldwide scale (FAO 2010), which means that they will not recover in the short and medium term. This second dimension has attracted considerable scientific attention and public concern in the last few years (Costello et al. 2008, Essington et al. 2006, Heal and Schlenker 2008, Pauly et al. 1998, Worm et al. 2006; 2009). From an economic perspective, the collapse of fish stocks is a problem over and above the inefficiently low stocks because many consumers like to consume a variety of different fish. A prominent example is the Japanese preference for sushi which is traditionally prepared from a whole variety of different species of fish and seafood.

In this paper we analyze the problem of overfishing with reference to both aspects, and we study the options available to multi-species fishery management. There is empirical evidence that different species of fish are imperfect substitutes, which means that consumers’ willingness to substitute one species for another is limited (Asche et al. 1997, Barten and Bettendorf 1989, Bose and McIlgorm 1996, Chiang et al. 2001, Fousekis and Revell 2005). This observation can be regarded as consumers having ‘preferences for diversity’. In other words, the more fish varieties (species) consumers buy and eat, the higher the utility they draw from fish consumption in total. We model such preferences for variety by drawing on the celebrated Dixit-Stiglitz utility function (Dixit and Stiglitz 1977). Stronger preferences for variety are thus represented by lower elasticity of substitution between the different products (here fish species). This means that a consumer who enjoys eating many varieties of fish is still willing to replace an expensive variety by a less expensive one, but he will do so to a lesser extent than a consumer who cares little about variety in fish consumption (and thus has a high elasticity substitution between different varieties).

In contrast to the preference structure we model the dynamics of the fish stocks in a very simple way. We employ a standard biomass growth model for each species and ignore any ecological interactions between different species as well as the age structure of fish stocks (Skonhoft et al. 2008; in press, Tahvonnen 2009). This enables us to focus on the demand-side interactions of different fish species and to
show how these interdependencies impact on harvest and, in extreme cases, may cause collapse of stocks.

We study four scenarios of multi-species fishery: (i) open access, (ii) socially optimal harvesting (and management) of all species, (iii) second-best optimal management of one species, taking into account spill-over effects of regulation on all other species fished under open access, and (iv) myopic management of one species ignoring the effects of regulation on other fish stocks.

We find that the stronger is the love of variety, the greater are the number of stocks prone to collapse. The intuition for this result is as follows. When a fish stock declines both marginal harvesting costs and thus the species’ market price increase. The higher the preference for variety, the less consumers will shift demand to more abundant and hence cheaper species and the less such demand shift contributes to relax the pressure on the dwindling stock. Thus, stronger preferences for seafood diversity may imply a decrease of fish biodiversity in the oceans. In particular we show that under open access a low elasticity of substitution in consumer preferences can trigger a whole cascade of collapsing fish stocks. This is remarkable since our model does not include any exogenous dynamics such as increasing demand or technical progress, frequently the driving force for collapse in other models (see for example Dasgupta and Heal 1979).

We further compare the three scenarios for fishery management with regard to the ‘strictness’ of regulation. For this purpose we apply two measures of strictness. Intuitively we would expect a stricter management to employ lower total allowable catches (TACs) or a higher harvesting fee. However, a lower TAC in the short term may lead to higher TACs in the long term steady-state. Hence, this measure of strictness can only be applied in a meaningful way to the initial state of regulating an overfished stock. When comparing the long-run effects of more or less strict regulation, the appropriate measure is the resulting long term stock size. A particular management is called stricter than another one when the resulting long term steady-state stock is higher. We employ both measures of strictness where appropriate and find that under symmetry assumptions (i) second-best optimal management of only one species must be less strict than socially optimal management of the same species, and (ii) myopic regulation of one species is too strict in comparison with second-best management. The intuitive reason is that regulation of one species drives up its price in the short term. This will increase demand for
the unregulated species. For those species with low resilience the resulting increase in harvesting pressure may have harmful effects. In the second-best scenario this negative spillover effect of regulation is taken into account and hence second-best management is less strict than first-best management of the same species. Myopic regulation, by contrast, is too strict as it ignores these effects. We present a numerical example in which myopic regulation of one species even leads to the collapse of stocks that would survive under full open access. Our overall conclusion is that the stronger are the preferences for variety, the more urgent is the need to coordinate regulation of different species (first-best management), or at least to account for regulation spill-overs (second-best management).

Our approach differs from the previous literature on multi-species fisheries which has mainly focused on biological interactions between different species (e.g., Conrad and Adu-Asamoah 1986, Lande et al. 2003, Quinn and Deriso 1999). Only few studies have looked at the economic value of diversity. Kasulo and Perrings (2006) focus on how biodiversity affects the effectiveness of fishing effort. They conclude that traditional freshwater fisheries in Malawi are associated with a higher diversity of fish catches than can be observed in profit-maximizing modern fisheries. Although not explicitly considering preferences for seafood diversity, a non-formal analysis by Wilson (1982; 1985) suggests that with relatively high cross-elasticities of demand for different species (in terms of our model, weak preferences for diversity) a multi-species fishery will be sustainable without any need for regulation (Wilson 1985:324). Our modeling analysis confirms this conjecture by showing that high elasticity of substitution favors the sustainability of the multi-species fishery. Another important strand of literature estimates demand (systems) for different fish varieties by using fish market data. While the elasticity of substitution between different fish species has not been estimated directly, several empirical studies show that different species of fish are generally imperfect substitutes. The degree of substitutability varies depending on how similar the species are, e.g. white fish or flat fish (Barten and Bettendorf 1989), and whether fish products are fresh, frozen, or otherwise processed (Chiang et al. 2001, Fousekis and Revell 2005). The general finding of fish species being imperfect substitutes gets wide empirical support: Asche et al. (1997) use data from European households; Barten and Bettendorf (1989) consider the demand for fish landed at Belgian sea ports; Fousekis and Revell (2005) analyze fish demand in Great Britain; Bose and McIl-
gorn (1996) and Chiang et al. (2001) study the Japanese demand for different species of tuna.

The remainder of this paper is organized as follows: In Section II we set up our theoretical multi-species fishery model with consumer preferences for seafood diversity. In Section III we study the impact of consumer preferences on open-access fisheries. Section IV deals with the socially optimal coordinated harvesting of all species. In Section V we analyze second-best management of a single species, taking into account the regulation spill-overs affecting other species, and in Section VI we discuss myopic management. In Section VII we illustrate some complex transitional dynamics by means of numerical simulations. The final section discusses the results and concludes.

II The Model

In the following we present the assumptions about the different species’ biomass growth, the harvesting technology, and consumer preferences.

Biomass growth of the different species

The stock dynamics of each species $i = 1, \ldots, n$ are mutually independent and described by a simple equation of motion for its biomass $x_i$

$$\dot{x}_{i,t} = g_i(x_{it}) - m_{it} h_{it},$$

(1)

where $g_i(x_i)$ is the biomass growth function, $m_{it}$ the mass of vessels, $h_{it}$ the harvest per vessel, and $H_{it} = m_{it} h_{it}$ the total harvest of species $i$, each at time $t$. All stocks grow according to the logistic function

$$g_i(x_{it}) = \rho_i x_{it} \left[1 - \frac{x_{it}}{\kappa_i}\right],$$

(2)

where $\rho_i$ is the intrinsic growth rate of species $i$ and $\kappa_i$ is the carrying capacity. In the remainder we will omit the time index $t$.

Harvest technology

Harvest is determined by a vessel’s fishing effort $e_i$ in catching species $i$ and by the stock size $x_i$. It is described by a generalized Gordon-Schaefer production function
\( h_i = h_i(x_i, e_i) = \nu_i x_i^\chi_i e_i^\epsilon_i, \)  

(3)

where \( \nu_i \) is the catchability coefficient of species \( i \) and \( \chi_i > 0 \) is the harvest elasticity of the stock of species \( i \). Returns on effort \( e_i \) are positive but decreasing, i.e. \( 0 < \epsilon_i < 1 \). Effort is measured in units of labor.\(^1\) Beside the variable fishing costs, there is a fixed cost \( \phi_i \) for operating a vessel capturing the capital cost of fishing (for vessel and fishing gear). Capital is perfectly malleable and can be used to harvest all species of fish, although the amount of capital needed may differ across species.

By assuming positive fixed costs, socially optimal harvesting and regimes of open access (or second-best or myopic management) differ in the mass of vessels, while the profit-maximizing current effort for targeting species \( i \) is socially optimal and determined only by its current price and its current stock size. Under decentralized decision-making fishermen take stock sizes \( x_i \), the wage rate \( \omega \), and output price \( p_i \) as given. A representative fisherman (but also a social planner) maximizes short-term profit per vessel \( p_i \nu_i x_i^\chi_i e_i^\epsilon_i - \omega e_i - \phi_i \) with respect to effort level \( e_i \). This results in:

\[ e_i = \left[ \frac{p_i \nu_i x_i^\chi_i}{\omega \epsilon_i} \right]^{1/(1 - \epsilon_i)} \]  

(4)

In appendices A.2 and A.9 we show that both under open access and in social optimum the harvesting costs per unit are given by \( c_i \ x_i^{-\chi_i} \) where

\[ c_i = \frac{\phi_i^{1-\epsilon_i} \omega^{\epsilon_i}}{(1-\epsilon_i)^{1-\epsilon_i} \epsilon_i^{\epsilon_i} \nu_i^{1-\epsilon_i}}. \]  

(5)

**Consumer preferences**

Consumer preferences over consumption of a numeraire commodity \((y)\) and consumption of fish \((v)\) are described by the utility function

\[ u(v,y) = \begin{cases} 
  y + \gamma \frac{y^{1/n}}{v^{1/n}} & \text{for } \eta \neq 1 \\
  y + \gamma \ln v & \text{for } \eta = 1.
\end{cases} \]  

(6)

\(^1\)When effort is measured in terms of mass of vessels, returns on effort may be increasing (Bjørndal and Conrad 1987). In our model, fishing effort and the mass of vessels are two separate variables.
The parameter $\gamma > 0$ describes the weight of fish consumption in utility, while $\eta > 0$ is the elasticity of demand for fish. Fish consumption is composed of different species, and the sub-utility of fish consumption is represented by the Dixit-Stiglitz (1977) utility function:

$$v = v(q_1, \ldots, q_n) = \left[ \sum_{i=1}^{n} q_i^{\frac{\sigma}{\sigma-1}} \right]^{\frac{\sigma-1}{\sigma}}, \quad (7)$$

where $q_i$ is the quantity of species $i$ consumed. The parameter $\sigma > 0$ measures the elasticity of substitution in the consumption of two different species. Higher values of $\sigma$ indicate varieties to be closer substitutes. A lower $\sigma$ can also be considered as a stronger preference for diversity. So high elasticity of substitution $\sigma$ can be associated with a consumer of fish fingers while low elasticity of substitution describes a consumer with a preference for sushi.

The representative household maximizes the utility function (6) with sub-utility (7) for fish subject to the budget constraint $\omega = y + \sum_{i=1}^{n} p_i q_i$, where $p_i$ is the price of species $i$. The first-order condition with respect to consumption of species $i=1, \ldots, n$ is given by

$$\gamma q_i^{\frac{1}{\sigma}} \left[ \sum_{j=1}^{n} q_j^{\frac{\sigma}{\sigma-1}} \right]^{\frac{\sigma-1}{\sigma}} = p_i, \quad (8)$$

Using $p = (p_1, \ldots, p_n)$ to denote the price vector and rearranging these conditions (see Appendix A.1), we can derive the demand function for each species $i$:

$$q_i(p) = \gamma^n p_i^{-\sigma} P^{\sigma-\eta}, \quad \text{where} \quad P = \left[ \sum_{j=1}^{n} p_j^{\frac{1}{1-\sigma}} \right]^{\frac{1}{1-\sigma}}, \quad (9)$$

is the price index for fish.

Obviously, demand for species $i$ is decreasing in its own price, as $\sigma > 0$. Since empirically different species of fish have been found to be (imperfect) substitutes

---

2Since $\ln v$ is the continuous extension of $\frac{\eta}{\eta-1} v^{\frac{\eta-1}{\eta}}$ for $\eta = 1$ there is no need to distinguish between $\eta \neq 1$ and $\eta = 1$ in the first-order conditions.

3The utility function given by equations (6) and (7) is the standard way of describing preferences for diversity, for example in industrial organization, international trade, and New Economic Geography. Equation (7) assumes identical elasticities of substitution for every pair of species. Pairwise different elasticities of substitution could be modeled by nested CES functions, which, however, makes the model less tractable does not yield additional insight.
(Asche et al. 1997, Barten and Bettendorf 1989, Fousekis and Revell 2005, Bose and McIlgorm 1996, Chiang et al. 2001), we concentrate on this case and assume $\sigma > \eta$. Under this condition, demand for fish species $i$ increases as prices for other species, aggregated in the price index $P$, go up. The condition $\sigma > \eta$ also implies that any fish variety can be better substituted by a different variety than by the numeraire commodity. Furthermore, we assume $\sigma > 1$, which means that the marginal utility of fish species $i$ is positive even if consumption of some other species $j \neq i$ is zero.\footnote{This assumption is well supported by empirical evidence: Asche et al. (1997, Table 4) report own-price and cross-price elasticities for salmon and crustaceans estimated from the EU trade data. Using their results we calculate an elasticity of substitution between fresh salmon and crustaceans of $\sigma = 1.66$ (cf. Appendix A.1).}

This assumption is important when we study collapse of some species, inducing zero consumption of that variety.

To close the model, we assume that each of the identical households inelastically supplies one unit of labor. The mass of households is normalized to unity such that total labor force is also equal to one. Rather than being employed in the fishery sector workers can produce the numeraire commodity with a constant returns to scale technology, where each unit of labor produces $\omega > 0$ units of output, i.e. the wage rate equals $\omega$. Given the effort levels required to catch each fish species and the fixed capital input for the fisheries, the output of the numeraire sector left for consumption is given by

$$ y = \omega \left[ 1 - \sum_{i=1}^{n} m_i \epsilon_i \right] - \sum_{i=1}^{n} m_i \phi_i. \quad (10) $$

Besides our general assumptions we employ additional more restrictive assumptions for some of the propositions derived below. The first one states that species are identical with respect to biological carrying capacities and harvesting technologies.

**Assumption 1.** *Species differ with respect to their intrinsic growth rates, w.l.o.g. we assume $\rho_1 < \rho_2 < \ldots < \rho_n$, but are symmetric otherwise, i.e. for all $i = 1, \ldots, n$, $\kappa_i = 1$, $c_i = c$, and $\chi_i = \chi < 1$. Furthermore, aggregate demand elasticity for fish is equal to one, i.e. $\eta = 1$.**
fish, as in this case aggregate expenditures for fish are constant and equal to $\gamma$ (see appendix A.1).

A second assumption states that species are completely symmetric with regard to their biological characteristics and the harvesting technologies employed.

**Assumption 2.** *Species are symmetric with regard to all parameters such that in addition to Assumption 1, growth rates are also the same, i.e. $\rho_i = \rho$.*

In the following sections we study different policy regimes, notably open access, socially optimal harvesting, second-best optimal management, and myopic management.

### III Open Access

Under open access, new vessels will enter the business until profits are driven to zero. Using the profit-maximizing effort level (4) and the corresponding harvest per vessel in that condition, we derive the open-access minimum average cost for species $i$ as a function of its stock. Since in equilibrium price must be equal to unit cost, we obtain

$$p_i = c_i x_i^{-\chi_i},$$  \hspace{1cm} (11)

where $c_i$ is given by (5). Equating supply ($= \text{total harvest } H_i = m_i h_i$) and demand \footnote{Equation (9) and then employing (11), we obtain} and then employing (11), we obtain

$$H_i = \gamma p_i^{-\sigma} \left[ \sum_{j=1}^{n} p_j^{1-\sigma} \right]^{-\frac{\sigma-1}{\sigma}} = \gamma^{\eta} \left[ c_i x_i^{-\chi_i} \right]^{-\sigma} \left[ \sum_{j=1}^{n} \left[ c_j x_j^{-\chi_j} \right]^{1-\sigma} \right]^{-\frac{\sigma-1}{\sigma}}. \hspace{1cm} (12)$$

Under Assumption 1 this expression simplifies to

$$H_i = \frac{\gamma}{c} \frac{x_i^{\chi} \sigma}{\sum_{j=1}^{n} x_j^{\chi} (\sigma-1)}. \hspace{1cm} (13)$$

Both in steady state and along the transitional dynamics, equation (12) (or 13) relates the total catch of species $i$ to the stocks of the different species. Since the different species are substitutes, the harvest of species $i$ will increase if the stock of some other species decreases.

Using (12) and (2) in (1), we derive the following differential equation describing...
the dynamics of stock $i$ under open access:

\[
\dot{x}_i = \rho_i x_i \left( 1 - \frac{x_i}{\kappa_i} \right) - \gamma_i \left[ c_i x_i^{-\chi_i} \right]^{-\sigma} \left[ \sum_{j=1}^{n} \left[ c_j x_j^{-\chi_j} \right]^{-1-\sigma} \right]^{-\frac{1}{\sigma}} - \sigma - \gamma_i \sigma^{-1}.
\] (14)

We first analyze the behavior of the multi-species fishery in open-access steady state. To obtain clear-cut results we employ Assumption 1:

**Lemma 1.** Under Assumption 1, steady-state stocks under open access are ordered according to intrinsic growth rates, $x_1^{oa} \leq x_2^{oa} \leq \ldots \leq x_n^{oa}$.

For the proof, see Appendix A.3.

We can use this lemma for the comparative statics analysis of the open-access steady-state stocks with regard to consumer preferences. The result is that a change in the elasticity of substitution $\sigma$ between the different species has a concertina-like effect on stock sizes.

**Proposition 1.** Let Assumption 1 hold. Then for all $\sigma$ there exists some species $i_0$ with $1 \leq i_0 < n$ such that such in the neighborhood of $\sigma$ we have:

\[
\frac{dx_i^{oa}}{d\sigma} > 0 \quad \text{for all } i < i_0
\]

\[
\frac{dx_i^{oa}}{d\sigma} \geq 0 \quad \text{for } i = i_0
\]

\[
\frac{dx_i^{oa}}{d\sigma} < 0 \quad \text{for all } i > i_0
\] (15)

For the proof, see Appendix A.4.

The result is illustrated in the top row of Figure 1 for a specific parameter set. An intuitive explanation for this result is that stronger preferences for diversity, implying lower values of $\sigma$, mean that consumers have a stronger desire to consume different species in similar proportions. This implies that harvest levels of different species are similar in size in the short term. In the mid and long term, this leads to diverging stocks, as under harvesting pressure the stocks of the less resilient species will decline faster than those of more resilient ones. If, by contrast, preferences for diversity are weaker, i.e. $\sigma$ increases, consumers are less willing to pay a high price for scarce species. The lower (higher) willingness to pay for relatively abundant (scarce) species induced by high values of $\sigma$ is also reflected by the open-access steady-state market prices as stated in the following result:

\footnote{Note that under Assumption 1 and with utility function (7) a quantitative stock size comparison for different species is meaningful as those have commensurable value for consumers.}
Figure 1: The steady-state stocks (left), prices (center), and harvests (right) of five species of fish with different intrinsic growth rates in the open-access steady state (top row) and in the socially optimal steady state (bottom row) for different elasticities of substitution $\sigma$. Intrinsic growth rates are $\rho_1 = 0.17$, $\rho_2 = 0.2$, $\rho_3 = 0.25$, $\rho_4 = 0.3$, and $\rho_5 = 0.4$; the other parameter values are $\kappa_i = 1$, $c_i = 1$, $\chi_i = 0.7$, for all $i = 1, \ldots, 5$, $\eta = 1$, $\gamma = 0.55$, $\delta = 0.1$. 
Corollary 1. Let Assumption 1 hold. Then for all $\sigma$ there exists some species $i_0$ with $1 \leq i_0 < n$ such that in the neighborhood of $\sigma$ we have:

\begin{align}
\frac{d p_{i_0}^{oa}}{d \sigma} &> 0 \quad \text{for all } i < i_0 \\
\frac{d p_i^{oa}}{d \sigma} &\geq 0 \quad \text{for } i = i_0 \\
\frac{d p_i^{oa}}{d \sigma} &< 0 \quad \text{for all } i > i_0
\end{align}

We now turn to the analysis of collapsing fish stocks. In the framework of our model, a fish species’ stock is defined to be collapsed (or depleted) if its stock size is zero.$^6$ Whether or not a species is prone to collapse depends on a simple condition on the harvesting function and the elasticity of substitution as stated in the following:

**Lemma 2.** Unless species $i$ is the only remaining species, $\chi_i \sigma > 1$ is a sufficient condition to guarantee that the open-access steady-state stock of species $i$ is strictly positive.

For the proof, see Appendix A.5.

Note that the condition above is always satisfied for the classical single-species Gordon-Schaefer model since in that case $\chi_i = 1$ and $\sigma = \infty$ (the substitution between different fish species is implicitly assumed to be perfectly elastic). However, for many species it is reasonable to assume $\chi_i < 1$. Table 1 provides a summary of some typical stock elasticities estimated for different species.

As argued in the introduction, also the elasticity of substitution between different species of fish is finite, $\sigma < \infty$. Using the value $\sigma = 1.66$ derived above (footnote 4) from Asche et al. (1997) and the values for $\chi_i$ from the literature (Table 1), condition $\chi_i \sigma > 1$ is violated for herring, pollock, prawn, anchovy, and yellow fin tuna. Under open access these species will be potentially fished down to collapse. As $\chi_i \sigma > 1$ is a sufficient but not a necessary condition for stocks to survive under open access, even these three species need not be depleted under that regime. If the above condition is violated, the affected species is still likely to survive under open access if the following conditions are satisfied: i) the species is

---

$^6$This definition is useful in the framework of our analytical model. A common definition used in empirical studies is that a stock is collapsed when harvest from this stock is below 10% of the historic maximum (Costello et al. 2008, Worm et al. 2006).
resilient, i.e. \( \rho_i \) and \( \kappa_i \) are high, ii) the harvesting cost parameter \( c_i \) is high, and iii) the weight of fish consumption in utility (\( \gamma \)) is low.

The next result (proved in appendix A.6) shows that if the stock of some species collapses, this will always happen in finite time rather than asymptotically.

**Proposition 2.** If the stock of some species collapses, it will do so in finite time.

In the following, we use \( T_i \) to denote the point in time when the stock of species \( i \) collapses. This collapse will be smooth in the sense that \( \dot{x}_i \to 0 \) as \( t \to T_i \). The collapse of stock \( i \) implies that both the harvest \( h_i \) per vessel and the number \( m_i \) of vessels fishing species \( i \) go to zero (see Appendix A.6). What happens to the harvest of the other species when the stock of species \( i \) collapses again depends on both the elasticity of substitution \( \sigma \) and species \( i \)'s stock elasticity \( \chi_i \).

**Proposition 3.** i) If \( \chi_i \sigma < (1 + \chi_i)/2 \), total harvest \( H_{jt} \) of all other species \( j \) with positive stock will rapidly increase to a new finite level with

\[
\lim_{t \to T_i} \frac{dH_{jt}}{dt} = +\infty.
\]

Total harvest \( H_{jt} \) of species \( j \) is continuous but not differentiable at \( t = T_i \).

ii) If \( \chi_i \sigma \geq (1 + \chi_i)/2 \), total harvest \( H_{jt} \) of all other species \( j \) with positive stock will behave smoothly at \( t = T_i \).

For the proof, see Appendix A.7.

The intuition for this result is as follows: The price of species \( i \) explodes as its stock collapses. If species are relatively poor substitutes (case i), harvest rates

<table>
<thead>
<tr>
<th>species/stock</th>
<th>value for ( \chi_i )</th>
<th>reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baltic cod</td>
<td>0.64</td>
<td>Kronbak (2005:472)</td>
</tr>
<tr>
<td>bigeye tuna</td>
<td>0.60</td>
<td>Grafton et al. (2007, online supporting material, p. 7)</td>
</tr>
<tr>
<td>North Sea herring</td>
<td>0.56</td>
<td>Bjørndal and Conrad (1987), Nøstbakken and Bjørndal 2003:351</td>
</tr>
<tr>
<td>pollock</td>
<td>0.50</td>
<td>Quinn and Deriso (1999:28)</td>
</tr>
<tr>
<td>Northern anchovy</td>
<td>0.39</td>
<td>Opsomer and Conrad (1994:29)</td>
</tr>
<tr>
<td>Australian Northern prawn</td>
<td>0.40</td>
<td>Grafton et al. (2007)</td>
</tr>
<tr>
<td>yellow fin tuna</td>
<td>0.23</td>
<td>Grafton et al. (2007)</td>
</tr>
</tbody>
</table>

Table 1: Stock elasticities for different fish species from the literature.
of the substitute species rise infinitely fast at the point in time when species $i$ is depleted. They approach new equilibrium levels on a market with one product less. By contrast, if it is sufficiently easy to substitute one species for another (case ii), the effect of the collapse of species $i$ will have a less pronounced effect.

These results indicate that strong preferences for diversity, i.e. low values of $\sigma$, tend to foster the collapse of fish stocks. Accordingly, the number of stocks surviving in the open-access steady state depends on $\sigma$. In the following proposition, we assume an initial open-access equilibrium with positive stock sizes for $n$ species when the elasticity of substitution is at some given level $\sigma > 1$. The proposition shows that a low elasticity of substitution may foster the collapse of the least resilient species. To derive a clear-cut result, we again employ Assumption 1.

**Proposition 4.** Let Assumption 1 hold for $n$ species in an initial open-access equilibrium at some given level of $\sigma$. If

$$ (n + 1 - i) \rho_i < \frac{\gamma}{c} \frac{(2 - \chi)^{2-\chi}}{(1 - \chi)^{1-\chi}} \quad \text{for species } i = 1, \ldots, n $$

(17)

a threshold value $\sigma$ ($1 < \sigma < \sigma$) for the elasticity of substitution exists such that the stocks of the $n$ ($1 \leq n \leq n$) least resilient species will sequentially collapse under open access fishing if the elasticity of substitution falls from $\sigma$ to a level equal to or below $\sigma$.

For the proof, see Appendix A.8.

Proposition 4 provides an explanation for the cascading collapse of species. If the elasticity of substitution falls to a sufficiently low value, the stocks of the least resilient species will collapse one after another under open access fishing. Those ‘least resilient’ species are characterized by intrinsic growth rates below threshold values specified by condition (17). Note that the lower the remaining number of fished stocks, the higher is the threshold value for the intrinsic growth rate.

A similar statement can be made for the case where species are identical with respect to ecological growth parameters ($\rho_i$ and $\kappa_i$) but differ with respect to harvesting technology, i.e. the cost parameters $c_i$. If species differ with respect to both the ecological parameters and harvesting technology, it will not be possible to derive a monotonic relationship between $\sigma$ and the number of surviving stocks. In other words, the equilibrium number of species may rise with stronger preferences for diversity (see also Section VII). The reason is that the effects of different
ecological characteristics and different harvesting technologies may go into opposite directions.

IV Social Optimum

We now consider socially optimal harvesting of all species. The social planner maximizes the representative household’s utility by simultaneously choosing intertemporally optimal harvesting paths for all species. Using $\delta$ to denote the social discount rate this problem can be written as (see Appendix A.9)

$$\max_{y, \{H_i\}} \int_{t=0}^{\infty} \left[ y + \gamma \frac{\eta}{\eta - 1} \left( \sum_{j=1}^{n} H_j^{\sigma} \right)^{\frac{\eta-1}{\eta}} - \frac{\eta}{\sigma-1} \right] \exp(-\delta t) dt$$  \hspace{1cm} (18)

subject to

$$\omega = y + \sum_{j=1}^{n} c_j \ H_j \ x_j^{-\chi_j} \hspace{1cm} \text{(numeraire sector)} \hspace{1cm} (19)$$

and (1), i.e. the fish stock dynamics for all species $i$.

The full set of conditions for first-best optimal management is given in Appendix A.10. For the optimal harvest of species $i$, we obtain

$$H_i = \gamma \pi_i^{-\sigma} \left( \sum_{j=1}^{n} \pi_j^{1-\sigma} \right)^{-\frac{\eta}{\sigma-1}}, \hspace{1cm} (20)$$

where $\pi_j$ is the shadow price for harvest of species $j$. In the optimal steady state this shadow price is given by (see Appendix A.10)

$$\pi_j = c_j \ x_j^{-\chi_j} + c_j \ x_j^{-\chi_j} \frac{\chi_j g_j(x_j)}{x_j \left[ \delta - g_j'(x_j) \right]}.$$  

The first term captures the marginal harvesting costs at a stock size $x_j$, corresponding to the market price of species $j$ in open access. The second term represents the marginal resource rents. In a decentralized economy this term also corresponds to the socially optimal harvesting fee or to the price for a quota in a competitive quota market if the total allowable catch is set at the socially optimal level.

Proposition 5. Socially optimal harvesting can be decentralized either by setting value-added harvesting fees for all $n$ species or by setting total allowable catches at the first-best harvesting quantities and issuing tradable harvesting quotas.
In steady state the optimal value-added harvesting fee (or the optimal price of the quota in percentage of marginal harvesting costs) for species $i$ is given by

$$\tau_i = \frac{\chi_j g_j(x_j)}{x_j [\delta - g'_j(x_j)]}.$$  \hfill (21)

There is an interesting parallel between the Dixit-Stiglitz models of monopolistic competition with preferences for diversity and the model considered here. Under monopolistic competition, firms set a price with a markup over marginal costs and thereby realize a monopoly rent. This markup leads to an inefficiency. Here, the markup $\tau_i$ over marginal harvesting costs equals the socially optimal harvesting fee. Setting this fee captures the resource rents dissipated when firms compete under conditions of open access.

If the species are symmetric except for different intrinsic growth rates (Assumption 1), the socially optimal steady-state stocks can be ordered in a similar way as under open access:

**Lemma 3.** Under Assumption 1, socially optimal steady-state stocks are ordered according to intrinsic growth rates, $x_1^* \leq x_2^* \leq \ldots \leq x_n^*$.

For the proof, see Appendix A.11.

Similar to the open-access regime a change in the elasticity of substitution has a concertina-like effect on the steady-state stock sizes. With stronger preferences for diversity the stock sizes of the less (more) resilient species, i.e. lower (higher) $\rho_i$, will decrease (increase).

**Proposition 6.** Let Assumption 1 hold. Then for all $\sigma$ there exists some species $i_0$ with $1 \leq i_0 < n$ such that in the neighborhood of $\sigma$ we have:

$$\frac{dx_i^*}{d\sigma} > 0 \quad \text{for all } i < i_0$$

$$\frac{dx_i^*}{d\sigma} \geq 0 \quad \text{for } i = i_0$$

$$\frac{dx_i^*}{d\sigma} < 0 \quad \text{for all } i > i_0$$

For the proof, see Appendix A.12.

For an illustration, see the bottom row of Figure 1. The intuition for the result is similar to the open-access regime. With stronger preference for diversity the marginal benefit for consumers is higher (smaller) for the species with low (high) intrinsic growth rates, which are relatively scarce (abundant) in the socially
optimal steady state (Lemma 3). Hence it is socially optimal that the species with low (high) intrinsic growth rates should be fished more (less) heavily, causing their steady-state stocks to decline (increase).

Further, both marginal harvesting costs and marginal resource rents decline with the stock size of the species under consideration.

**Corollary 2.** Let Assumption 1 hold. Then for all \( \sigma \) there exists some species \( i_0 \) with \( 1 \leq i_0 < n \) such that in the neighborhood of \( \sigma \) we have:

\[
\begin{align*}
\frac{d\tau_i}{d\sigma} &< 0 \quad \text{for all } i < i_0 \\
\frac{d\tau_i}{d\sigma} &\leq 0 \quad \text{for } i = i_0 \\
\frac{d\tau_i}{d\sigma} &> 0 \quad \text{for all } i > i_0
\end{align*}
\]

In other words, the stronger consumers’ preference for diversity, the lower (higher) the optimal harvesting fee for the more (less) resilient species.

With sufficiently strong preferences for diversity, the collapse of the less resilient species may be socially optimal, provided that the intrinsic growth rate of such a species is smaller than society’s discount rate. More precisely, we can state:

**Lemma 4.** Under first-best management, the steady-state stocks of all species with \( \chi_i \sigma > 1 \) (provided species \( i \) is not the only remaining species) or \( \rho_i > \delta \) are strictly positive.

For the proof, see Appendix A.13.

Thus one of two independent conditions is sufficient to guarantee a strictly positive steady-state stock under first-best harvesting. The first condition implies that it cannot be optimal to deplete a stock that would not be depleted under open access (see Lemma 2). The second condition states that it cannot be optimal to deplete a stock that has an intrinsic growth rate greater than the social discount rate. Note also that Lemma 4 provides sufficient but not necessary conditions for stocks to be strictly positive in social optimum.

According to Lemma 2 for the open-access regime and Lemma 4 for the social optimum, it is possible to provide lower bounds for steady-state stock numbers both in the open-access regime and in social optimum. We define:

- \( n_{oa}^{\text{min}} \) as the number of species satisfying \( \chi_i \sigma > 1 \) (cf. Lemma 2),
- \( n_{min}^{*} \) as the number of species satisfying \( \chi_i \sigma > 1 \) or \( \rho_i > \delta \) (cf. Lemma 4).
Both lower bounds weakly increase whereas the difference between those two weakly decreases with $\sigma$. This result, formally stated below (and illustrated in Section VII), suggests that stronger preferences for diversity enhance the need for fishery management.

**Proposition 7.** $n^*_\min - n^{oa}_\min$ weakly decreases with $\sigma$.

For the proof, see Appendix A.14.

V  Second-best Optimal Management

It is obviously difficult to fully control the worldwide harvesting activities for all species interacting on the market. Evidence (e.g. from European fishery policy) shows that fishing regulations often refer to single species that are under particular pressure. It is therefore useful to study scenarios where a regulator can only control a limited number of species. In such a case the regulator should however take into account the spill-over effects of regulation on the harvest and the stock dynamics of other non-regulated species. For simplicity we will only consider the case where the regulator can control just one species, indexed by $k$. Our results generalize in a straightforward way to controlling several but fewer than all species.

To keep the analysis tractable we assume that the non-managed species $i \neq k$ are harvested under open-access and their harvest quantities are determined by inverse demand (8) and the open-access condition (11), i.e.

$$\gamma q_i^{-\frac{1}{\sigma}} \left[ q_k^{\frac{\eta-1}{\sigma}} + \sum_{j \neq k} q_j^{\frac{\eta-1}{\sigma}} \right]^{\frac{n-1}{\eta} \frac{\sigma}{\sigma-1} - 1} = c_i x_i^{-\chi_i} \text{ for all species } i \neq k. \quad (24)$$

If all markets clear, i.e. $q_i = H_i$, the regulator’s optimization problem in the second-best management setting reads as follows:

$$\max_{y, \{H_i\}} \int_{t=0}^{\infty} \left[ y + \gamma \frac{\eta}{\eta - 1} \left[ H_k^{\frac{\eta-1}{\sigma}} + \sum_{j \neq k} H_j^{\frac{\eta-1}{\sigma}} \right]^{\frac{n-1}{\eta} \frac{\sigma}{\sigma-1} - 1} \right] \exp(-\delta t) \, dt \quad (25)$$

subject to (1), (19), and (24) with $q_i = H_i$. The optimality conditions for this problem are given in Appendix A.15 (assuming an interior optimum).

We will now compare first- and second-best optimal management rules. Intuition suggests that regulation of one species drives up its price (at least in the
short run) such that the demand for the unregulated species will increase. If resilience of the unregulated species is low, too strict regulation may have harmful effects. Accordingly, it is fair to conjecture that second-best optimal regulation of one species is less strict than first-best optimal regulation of all species.

Both transitional dynamics and steady-state levels will be different under socially optimal and second-best optimal management. One way of directly comparing the two regimes would be to consider the shadow prices in the initial state when stock sizes are the same. This, however, would require fully solving both dynamic optimization problems, which is too complicated to generate meaningful insights. Under the symmetry Assumption 2, however, it is possible to compare the steady-state levels for the two regimes. For this comparison we consider one management rule to be stricter than another rule if it leads to a higher steady-state stock of the managed species.

**Proposition 8.** Let Assumption 2 hold. If \( \rho > \delta \) and

\[
\frac{c}{1 + \frac{\chi}{2\delta}} < 2^2 \chi \gamma \left[n \rho \left[1 + \frac{\chi}{2\delta}\right]\right]^{-1}
\]

holds, the second-best optimal steady-state stock of the managed species \( x_{sb}^k \) will be smaller than the socially optimal steady-state stock, i.e. \( x_{sb}^k < x^*_k \). The second-best optimal steady-state stocks of the non-managed species \( x_{sb}^i \) for \( i \neq k \) lie between the steady-state levels in the open-access regime and the socially optimal stock, i.e. \( x_{oa} < x_{sb}^i < x_{sb}^k < x^* \) for all \( i \neq k \).

For the proof, see Appendix A.16.

This result confirms the intuition described above. Since the other species are substitutes for the regulated species, the reduced supply of the managed species leads to increased demand for and thus to rising fishing pressure on the non-managed species. In order to avoid an overall welfare-reducing effect, second-best optimal management must therefore be less strict than first-best management.

Note that condition (26) is sufficient but not necessary for Proposition 8 to hold. Condition (26) implies that steady-state harvest under first-best management exceeds that under open access, i.e. \( g(x^*) \geq g(x_{oa}) \). More precisely, when (26) holds, the socially optimal steady-state stock is smaller or equal to the stock size that generates the maximum sustainable yield. This is the case if the discount rate is sufficiently high, the intrinsic growth rate is sufficiently low and the harvesting costs are sufficiently low.
VI  Myopic Management

In the final scenario we study the effects of myopic regulation. A regulator is said to be myopic when he ignores that managing some species \( k \) may affect other species being harvested under open access. The optimization problem is similar to that of second-best optimal management. It is formally identical to (25) except that the regulator ignores constraint (24). The resulting myopic optimality conditions are given in Appendix A.17. Note that under myopic management, the regulator will have to re-optimize continuously as prices of the other species will not stay constant contrary to the myopic regulator’s expectation.

At the given initial stock sizes second-best and myopic management can be directly compared with regard to harvest levels as in both cases only one species is managed while all other species are harvested in a regime of open access. Thus, we can use the initial total allowable catches (TACs) or, equivalently, the initial harvesting fees to evaluate the strictness of regulation here: a management rule is stricter than another one if the initial TAC is lower or the initial harvesting fee is higher. We find that myopic management is too strict compared to second-best management.

**Proposition 9.** *The myopically optimal harvesting fee is larger than the second-best optimal harvesting fee.*

For the proof, see Appendix A.18.

VII  Numerical Illustrations

In this section we present the results of some numerical simulations to show the dynamic patterns that can arise under open access and management scenarios.

*Preferences for diversity and collapse of fish stocks*

If the supply of one species decreases, the demand for the other species will increase as the different species are substitutes in consumption. One reason for the decreasing supply of one species may be that its stock is about to collapse. Since the price mechanism boosts demand for the other species, the collapse of one stock may induce a collapse of other stocks too.
Figure 2: The stocks (left column), prices (center column), and harvest rates (right column) for five fish species with different intrinsic growth rates. Choice of parameters: \( \kappa_i = 1, c_i = 1, \chi_i = 0.44, \) for all \( i = 1, \ldots, 5, \) \( \eta = 1, \) \( \rho_1 = 0.22, \) \( \rho_2 = 0.26, \) \( \rho_3 = 0.33, \) \( \rho_4 = 0.45, \) \( \rho_5 = 0.7. \) top row: \( \sigma = 2, \gamma = 0.5; \) middle row: \( \sigma = 1.2, \gamma = 0.5; \) bottom row: \( \sigma = 2, \gamma = 0.6. \)
Figure 2 shows the simulation results\textsuperscript{7} of the open-access dynamics. Here we study a multi-species fishery with five fish stocks characterized by different intrinsic growth rates but identical parameter values otherwise. The three columns show the development of stocks (left column), prices (center column), and harvest rates (right column) for three parameter sets differing in the elasticity of substitution ($\sigma$) and the weight of fish consumption in utility ($\gamma$).

The three graphs in the top row of Figure 2 show the time paths of stocks, prices, and harvest rates for the case of weak preferences for diversity ($\sigma = 2$) and a relatively low weight of fish consumption in utility ($\gamma = 0.5$). Starting at a steady state without fishing, all stocks first decline, then reach positive, though not optimal steady-state levels. The prices for all species increase as the marginal harvesting costs increase with declining stock sizes. Harvest rates for all species initially decline, but the harvest rate of species 5 reaches an steady-state level above its initial value.

In the three graphs in the middle row of Figure 2 we display the corresponding time paths for the case of strong preferences for diversity ($\sigma = 1.2$) and $\gamma = 0.5$ as above. Condition (17) is fulfilled for all species $i = 1, \ldots, 5$ in this example and thus Proposition 4 states that there is a threshold value $\sigma$ such that all the stocks will collapse if the elasticity of subsitution falls below this threshold value. For $\sigma = 1.2$ this is the case. We see that the least resilient species (the one with the lowest $\rho_i$) suffers a serious decline in stock while initially all the other species seemingly approach positive steady-state values. However, once the least resilient species has been depleted, the stock of the species with the second-lowest growth rate also starts to decline sharply. After that stock has also collapsed, the same thing happens to the species with the next-lowest growth rate, and so forth. Along with a collapsing stock, both the corresponding marginal harvesting cost and price explode to infinity. Close to the point in time where species 1 approaches depletion the harvest rates of all four remaining species sharply increase. This illustrates part (i) of Proposition 3. For the parameter values used here, the condition $\chi_i \sigma < (1 + \chi_i)/2$ is fulfilled (as $\chi \sigma = 0.53$ and $(1 + \chi)/2 = 0.72$).

The three graphs in the bottom row of Figure 2 show the corresponding time paths for a set of parameters satisfying condition $\chi_i \sigma > (1 + \chi_i)/2$ in part (ii) of

\textsuperscript{7} All simulations were done with MATLAB R2010b. Codes are available from the authors on request.
Proposition 3. Here, preferences for diversity are weak ($\sigma = 2$), but the weight of fish consumption in utility is slightly higher as in the example before ($\gamma = 0.6$). Stocks collapse under open-access fishing, but the harvest rates of the remaining species behave smoothly at the points in time where the other species collapse.

In the examples shown in the middle and bottom rows of Figure 2, all stocks collapse under the open-access regime. For a different parameter specification (in particular if condition (17) is fulfilled for the less resilient, but not for the more resilient species) we can obtain both, some stocks being depleted while other stocks reach positive steady-state levels even under open-access fishery.

Preferences for diversity in the open-access setting may thus provide an explanation for the cascading collapse of fish stocks that has recently been documented on a worldwide scale (Costello et al. 2008, Heal and Schlenker 2008, Worm et al. 2006; 2009). This is perhaps surprising as the model does not include any exogenous dynamic driving forces such as technical progress in fishing technology or increasing overall demand for fish, which are also likely to contribute to the collapse observable in reality (Skonhoft 2009, Squires and Vestergaard 2009).

To assess the quantitative effect of a change in $\sigma$ on the number of stocks that survive under open access and under first-best management we conduct a Monte-Carlo-like simulation. Using the Latin Hypercube sampling method we randomly and independently selected values for the three parameters $\rho_i$, $c_i$, $\chi_i$ for $n = 33$ species according to uniform distributions, while $\kappa_i = 1$ for all $i$, $\eta = 1$, $\gamma = 0.66$, and $\delta = 33\%$ are fixed. The intrinsic growth rates are drawn from the interval $\rho_i \in [0.1; 1.5]$, the cost parameters from $c_i \in [0.75; 1.25]$, and the stock elasticities from $\chi_i \in [0.25; 1]$. Then by varying $\sigma$ we determine the number of species that have strictly positive steady-state stocks, first in open access and then under socially optimal management.

In the left hand diagram of Figure 3 the thick solid (thick dotted) line depicts the steady-state number of stocks as a function of $\sigma$ under open access (under first-best management). The thin lines depict the corresponding lower bounds $n_{\min}^{oa}$ and $n_{\min}^{*}$. For the calculation a randomly chosen set of parameters is held fixed (except for $\sigma$) and for each $\sigma \in [1.1, 3]$ the steady-state numbers of species with strictly positive stock sizes (and their lower bounds) are calculated. We observe that the

---

8We have chosen a comparatively high discount rate of $\delta = 33\%$ in order to trigger the collapse of some stocks in a social optimum.
Figure 3: The simulated number of stocks in the open-access steady state (thick solid lines) and the first-best optimal steady state (dotted lines) as a function of $\sigma$ for one parameter set (diagram left) and averaged over a random sample of 50 parameter sets (diagram right).

The steady-state number of stocks under open access does not increase monotonically in $\sigma$. The steep downward decline at $\sigma = 1.24$ is due to the different effects of the various parameters discussed at the end of Section III.

The right-hand diagram in Figure 3 shows the result of averaging these curves over 50 such parameter sets. We see that the relationship between $\sigma$ and the steady-state number of species becomes almost monotonic, both under open access and under optimal management. The two thin lines depicting the averaged lower bounds $n_{\text{min}}^*$ and $n_{\text{oa min}}^*$ are weakly increasing with $\sigma$, and their difference decreases with $\sigma$ (cf. Proposition 7).

Comparison of fishery management scenarios

In the following, we compare the different management scenarios using a numerical example with species satisfying symmetry Assumption 1. The intrinsic growth rates are chosen as $\rho_1 = 0.5$, $\rho_2 = 0.52$, $\rho_3 = 0.56$, $\rho_4 = 0.6$, and $\rho_5 = 0.66$. Moreover, we select a stock elasticity of harvest $\chi = 0.33$, a discount rate $\delta = 0.2$, and an elasticity of substitution between species of $\sigma = 2$.

Read from top to bottom, Figure 4 shows the simulation results for (i) open access, (ii) socially optimal harvesting, (iii) second-optimal management, and (iv)
Figure 4: Stocks (left column), (shadow)prices (center column), and harvest rates (right column) for five species under open access (top row), socially optimal management (second row from top), second-best management of species 3 (third row from top), and myopic management of species 3 (bottom row). Parameter values: $\kappa_i = 1$, $c_i = 1$, $\chi_i = 0.33$, for all $i = 1, \ldots, 5$, $\eta = 1$, $\gamma = 0.88$, $\delta = 0.2$, $\sigma = 2$, $\rho_1 = 0.50$, $\rho_2 = 0.52$, $\rho_3 = 0.56$, $\rho_4 = 0.60$, and $\rho_5 = 0.66$. 
myopic management. For the five different species, the left-hand column displays the time paths of the stocks, the central column those of prices (in the case of open-access fishing) and shadow prices (in case of the management scenarios) while the right hand column contains those of the harvesting rates.

Under open access, stock sizes start at the carrying capacities. Similarly to the top row in Figure 2, stock sizes decline and prices increase and eventually converge to the stable steady-state levels. For the three management scenarios, we assume that regulation starts off at open-access steady-state stock levels. Socially optimal harvest quantities (as control variables) start at low levels and then increase. Accordingly, shadow prices (as co-state variables) start at high levels and then decrease. As a consequence stocks (as state variables) recover and reach steady-states well above the maximum sustainable yield levels. The latter observation holds because larger stock sizes induce lower harvesting costs. Note also that in all scenarios the optimal steady-state shadow prices exceed the open-access prices.

Under second-best management (third row) we assume that species 3 is managed while the remaining species 1, 2, 4, and 5 are harvested under open access. Harvesting of the controlled species initially also starts at a lower level than the open-access equilibrium harvest. This leads to an increase of the managed stock. Since species are substitutes, harvests of the non-managed species slightly increase, and the corresponding stocks slightly decline. However, as the steady-state stock and harvest of species 3 are higher under second-best management than under open access, harvest levels for the non-managed species slightly decline, and the steady-state stocks are also modestly above the initial open-access steady-state levels. Overall, the changes in stock and harvest of the managed species are much smaller under second-best management than in the social optimum. This pattern reflects our analytic result from Proposition 8, establishing that second-best optimal management is less strict than first-best optimal management. We find this also for the case shown in the figure where species differ with respect to their intrinsic growth rates.

Under myopic management (bottom row of Figure 4), harvesting of the regulated species 3 starts at a drastically lower level than the initial levels under first- or second-best management. This induces the harvest rate of all other species to jump to an initially high level because consumers replace the expensive regulated species with cheaper non-regulated species, including those species with
small intrinsic growth rates. Eventually, the harvest rates of the non-managed species decline. The increased harvesting pressure on the least resilient species leads to its collapse, reinforcing the harvesting pressure on the remaining species. This ultimately leads to the collapse of all species except for the managed one, which finally reaches a steady-state level below the initial open-access level. The reason for the latter, rather disturbing phenomenon is that at the end of the day the managed fishery has to supply the whole market. This example demonstrates strikingly that regulation can cause quite detrimental effects on other species not subject to regulation. The channel of such unwanted side-effects runs through the demand-side feedback of consumers who replace the regulated product with other products. At first glance, this observation may suggest that high elasticity of substitution (preference for fish fingers) is bad. However, the contrary is the case. If the elasticity of substitution is high, the consumer will eat fish fingers consisting of mainly abundant species. If the elasticity of substitution is low, they will suffer from decreasing product variety and will want to substitute the regulated expensive product with a whole set of other varieties, including those produced from the less resilient species. This example shows that it is important to account for spillovers of regulation on other, possibly even more threatened species.

**VIII Discussion and Conclusions**

We have analyzed the impact of consumer preferences for seafood diversity on harvesting patterns and stock dynamics by means of a multi-species fishery model. Our results open up a new perspective on the economics of collapsing fish stocks and on multi-species fishery management. We have shown that consumer preferences for diversity may play an important role in the ongoing collapse of fish stocks presently observed.

Clearly other factors are likely to also contribute to the collapse of fish stocks. In particular technical progress and exogenously changing consumer patterns may increase harvesting pressure (Skonhoft 2009, Squires and Vestergaard 2009). In contrast to these explanations for the collapse of fish stocks, our analysis highlights the fact that different fisheries are linked through demand-side interactions, and we have derived some important consequences for management.

We have shown that, other things being equal, species with low intrinsic growth
rates are likely to collapse first under a regime of open access. As high-trophic level species typically have lower growth rates than low-trophic ones (Froese and Proelß 2010), this result provides an explanation for the empirical observation that mean trophic level of catches has declined over time ("fishing down the food web", Pauly et al. 1998, Essington et al. 2006). Specifically, our model suggests that a sequential collapse and replacement of higher-trophic-level species by lower-trophic-level ones will occur. Essington et al. (2006) identify a second mechanism, where the mean trophic level of catches decreases by the sequential addition of less valuable, lower-trophic-level species ("fishing through the food web"). This mode of fishing down the food web could be explained by a slightly modified version of our model where consumers value species differently and overall fishing pressure increases, e.g. due to a generally increasing demand for fish or due to technical progress in fishing technology. A full analysis of this issue is beyond the scope of this paper, however.

Recent empirical studies have argued that management by individual transferable quotas (ITQs) could significantly reduce the global trend toward the widespread collapse of fish stocks (Costello et al. 2008, Heal and Schlenker 2008). However, these studies have mainly focused on the own-stock effects of the regulated species. They have not tested for possible negative spill-overs onto other stocks. This would however be important, since our analysis has shown that myopic management of some stocks may cause the collapse of other non-regulated stocks, even though the managed stock itself does not collapse. Indeed, fishing effort has moved from industrialized countries (with relatively strict regulation of their fisheries) to developing countries (especially African countries with less strict regulation and enforcement), thus increasing harvesting pressure in those countries (Worm et al. 2009).

One important conclusion from our analysis is that in order to prevent the world-wide collapse of fish stocks, it is necessary to move from the current management of some stocks to a coordinated management of all fish stocks. If first-best coordinated management of all species is not possible, spill-overs to other non-regulated species should be taken into account in order to prevent the unwanted collapse of those.

9 We thank a reviewer for pointing this out.

10 To model this, weights could be introduced into the utility function (7) such that utility from fish consumption would be $v(q_1, \ldots, q_n) = \left[ \sum_i \alpha_i q_i^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}$. 

28
Appendix

A.1 Derivation of Demand Function (9)

Taking both sides of condition (8) to the power of $1 - \sigma$ and summing over $i$, we obtain

$$\sum_{i}^{n} p_{i}^{1-\sigma} = \left[ \gamma v^{\frac{n-1}{\sigma}}\sigma^{-1} \right]^{1-\sigma} \sigma^{-1} = \left[ \gamma v^{-\frac{1}{\sigma}} \right]^{1-\sigma}$$  \hspace{1cm} (27)

$$\Leftrightarrow \quad v = \gamma^{\eta} \left[ \sum_{i}^{n} p_{i}^{1-\sigma} \right]^{-\frac{1}{1-\sigma}} = \gamma^{\eta} P^{-\eta}$$  \hspace{1cm} (28)

where $P$ is the price index for fish. Note that for $\eta = 1$ overall expenditures for fish are constant and equal to $\gamma$. Substituting into (8) and rearranging leads to the demand functions (9). Own and cross elasticity of demand (9) are

$$\frac{p_{i}}{q_{i}} \frac{\partial q_{i}}{\partial p_{i}} = -\sigma + (\sigma - \eta) \frac{p_{i}^{1-\sigma}}{\sum_{j=1}^{n} p_{j}^{1-\sigma}}$$  \hspace{1cm} (29)

$$\frac{p_{i}}{q_{j}} \frac{\partial q_{j}}{\partial p_{i}} = (\sigma - \eta) \frac{p_{i}^{1-\sigma}}{\sum_{j=1}^{n} p_{j}^{1-\sigma}}$$  \hspace{1cm} (30)

Hence,

$$\sigma = \frac{p_{i}}{q_{j}} \frac{\partial q_{j}}{\partial p_{i}} - \frac{p_{i}}{q_{i}} \frac{\partial q_{i}}{\partial p_{i}}$$  \hspace{1cm} (31)

If we take crustaceans as species $i$ and fresh salmon as species $j$, and use the compensated price elasticities from Asche et al. (1997, Table 4), we obtain $\sigma = 0.898 - (-0.762) = 1.66$.

A.2 Cost of Fishing under Open Access

Using (4) the zero-profit condition $p_{i} \nu_{i} x_{i}^{\chi_{i}} e_{i}^{\epsilon_{i}} - \omega e_{i} = \phi_{i}$ may be written as

$$e_{i} = \frac{\epsilon_{i}}{1-\epsilon_{i}} \frac{\phi_{i}}{\omega}$$  \hspace{1cm} (32)

Substituting into the harvesting function (3) we derive equilibrium harvest as a function of the species’ biomass: $h_{i} = \nu_{i} x_{i}^{\chi_{i}} \left[ \frac{\epsilon_{i}}{1-\epsilon_{i}} \frac{\phi_{i}}{\omega} \right]^{\epsilon_{i}}$. Total cost of fishing is $\omega e_{i} + \phi_{i} = \phi_{i} e_{i}/(1 - \epsilon_{i}) + \phi_{i} = \phi_{i}/(1 - \epsilon_{i})$. Hence cost per unit of harvest may be written as $c_{i} x_{i}^{-\chi_{i}}$, where $c_{i}$ is given by (5).
A.3 Proof of Lemma 1

Under Assumption 1 total harvest $H_i^{oa}$ of species $i$ is given by equation (13). For the stable equilibrium stock, open-access harvest must increase more strongly with stock size than natural growth, i.e. the stable equilibrium is characterized by the condition

$$\frac{dH_i^{oa}}{dx_i^{oa}} > \rho_i \left[ 1 - 2 \frac{x_i^{oa}}{\kappa} \right] \tag{33}$$

Next, we differentiate the equilibrium condition $\rho_i x_i^{oa} \left[ 1 - \frac{x_i^{oa}}{\kappa} \right] = H_i^{oa}$ with respect to $\rho_i$, where $H_i^{oa}$ as given by (13) is a function of the stock sizes. This leads to the condition

$$x_i^{oa} \left[ 1 - \frac{x_i^{oa}}{\kappa} \right] = -\rho_i \left[ 1 - 2 \frac{x_i^{oa}}{\kappa} \right] + \frac{dH_i^{oa}}{dx_i^{oa}} \frac{dx_i^{oa}}{d\rho_i}$$

By condition (33), the term in brackets on the RHS of this condition is positive. We thus have $dx_i^{oa}/d\rho_i > 0$, i.e. the larger the intrinsic growth rate, the larger the steady-state stock.

A.4 Proof of Proposition 1

In a first step, we prove the following intermediate result: For all $\sigma^{oa}$ there exists a species $i_0$ such that if we increase $\sigma$ in a neighborhood of $\sigma^{oa}$, the harvest of all species $i \leq i_0$ at given stock sizes will decrease. Differentiating (13) with respect to $\sigma$ leads to

$$\frac{\partial H_i}{\partial \sigma} = H_i \chi \left[ \ln x_i - \frac{\sum_{j=1}^{n} x_j^{\chi(\sigma-1)} \ln x_j}{n} \right] \tag{34}$$

The last term in brackets is independent of the species under consideration. It is the logarithm of a weighted geometric mean of fish stocks:

$$\frac{\sum_{j=1}^{n} x_j^{\chi(\sigma-1)} \ln x_j}{\sum_{j=1}^{n} x_j^{\chi(\sigma-1)}} = \ln \left[ \prod_{j=1}^{n} x_j^{\chi(\sigma-1)} \right] \tag{35}$$

Since the logarithm of the stock size of some species must be smaller than the weighted geometric mean, the RHS of (34) must be negative for those species. Now let $i_0$ denote the species with the largest stock smaller than the weighted
geometric mean. For all species \(i\) with \(i \leq i_0\), the RHS of equation (34) is still negative, except for the case where the bracket term in (35) is zero, in which case harvest of species \(i_0\) does not change with \(\sigma\).

Differentiating the equilibrium condition \(\rho_i x_i (1 - x_i / \kappa_i) = H_i\) with respect to \(\sigma\) yields

\[
\left[ \rho_i \left[ 1 - 2 \frac{x_i^{oa}}{\kappa_i} \right] - \frac{dH_i^{oa}}{dx_i^{oa}} \right] \frac{dx_i^{oa}}{d\sigma} = \frac{\partial H_i^{oa}}{\partial \sigma}
\]

As the term in brackets on the left hand side of this equation is negative (see Appendix A.3) we conclude that for or all \(\sigma^*\) there exists a species \(i_0\) such that an increase of \(\sigma\) in a neighborhood of \(\sigma^*\) raises the open-access equilibrium stocks of all species \(i < i_0\).

A.5 Proof of Lemma 2

We show that for a sufficiently small initial stock size, \(x_i\) will increase under open access. For this purpose we show that under the conditions of proposition 1 growth \(g_i(x_i) - m_i h_i\) increases with \(x_i\) at \(x_i = 0\). Using (12), we get

\[
\lim_{x_i \to 0} \frac{\partial}{\partial x_i} \left\{ \rho_i x_i \left[ 1 - \frac{x_i}{\kappa_i} \right] - \gamma^n [c_i x_i^{\chi_i}]^{-\sigma} \left[ \sum_{j=1}^n [c_j x_j^{\chi_j}]^{1-\sigma} \right]^{-\frac{\sigma-n}{\sigma-1}} \right\}
\]

\[
= \lim_{x_i \to 0} \left\{ \rho_i \left[ 1 - 2 \frac{x_i}{\kappa_i} \right] - \chi_i \sigma \gamma^n c_i\sigma x_i^{\chi_i} \left[ \sum_{j=1}^n [c_j x_j^{\chi_j}]^{1-\sigma} \right]^{-\frac{\sigma-n}{\sigma-1}} \right. \\
- \left. \chi_i (\eta - \sigma) \gamma^n c_i^{-\sigma} x_i^{2 \chi_i} \chi_i^{\sigma-1} \left[ \sum_{j=1}^n [c_j x_j^{\chi_j}]^{1-\sigma} \right]^{-\frac{\sigma-n}{\sigma-1}} \right\}
\]

\[
\geq \lim_{x_i \to 0} \left\{ \rho_i \left[ 1 - 2 \frac{x_i}{\kappa_i} \right] - \chi_i \sigma \gamma^n c_i^{-\sigma} x_i^{\chi_i} \chi_i^{\sigma-1} \left[ c_j x_j^{\chi_j} \right]^{\eta-\sigma} \right\} = \rho_i > 0 \quad (36)
\]

A.6 Proof of Proposition 2

Consider first the case where some species will never collapse. Define

\[
\underline{x}_j \equiv \liminf_{t \geq 0} \{x_{jt}\}
\]

with \(\underline{x}_j > 0\) for some species \(j\). Then define the quantity

\[
\Omega \equiv \gamma^n \left[ \sum_{j=1}^n c_j^{1-\sigma} \underline{x}_j^{\chi_j (\sigma-1)} \right]^{-\frac{\sigma-n}{\sigma-1}} > 0
\]
Thus the equation of motion for species $i$ (14) can be bounded from above:

$$\dot{x}_{it} = \rho_i x_{it} \left[ 1 - \frac{x_{it}}{\kappa_i} \right] - \gamma^\eta \left[ c_i x_{-\chi_i}^\eta \right]^{1-\sigma} \left[ \sum_{j=1}^n \left( c_j x_j^{-\chi_j} \right)^{1-\sigma} \right]^{-\frac{\sigma-\eta}{\sigma-1}}$$

$$< \rho_i x_{it} - \Omega c_i^{-\sigma} x_{it}^{\sigma \chi_i} \equiv \frac{d\tilde{x}_{it}}{dt}$$

The solution to the last differential equation is given by

$$\tilde{x}_{it} = \begin{cases} 
\frac{\Omega}{\rho_i c_i} \left( 1 - e^{\rho_i (1-\sigma \chi_i) (1-\tilde{T}_{T_i})} \right) \left( 1 - \sigma \chi_i \right) & \text{for } t \leq \tilde{T}_{T_i} \\
0 & \text{for } t > \tilde{T}_{T_i} 
\end{cases}$$

(37)

where $\tilde{T}_{T_i} > 0$, the point in time after which $\tilde{x}_i = 0$, is a constant of integration. Since the true function $x_{it}$ falls faster than $\tilde{x}_{it}$, the stock of species $i$ will approach zero at a point in time $T_i < \tilde{T}_{T_i}$. Consider now the case where all stocks collapse and look at the species that converges to zero with the lowest speed. Then all other species must collapse in finite time by virtue of the first case. Therefore, after the point in time when the second last species has collapsed, (14) reduces to

$$\dot{x}_{it} = \rho_i x_{it} \left[ 1 - \frac{x_{it}}{\kappa_i} \right] - \gamma^\eta c_i^{-\eta} x_{it}^{\chi_i \eta \chi_i} < \rho_i x_{it} - \gamma^\eta c_i^{-\eta} x_{it}^{\chi_i \eta} \equiv \frac{d\tilde{x}_{it}}{dt}$$

Since species $i$ can only collapse if $\chi_i \sigma < 1$ and since $\sigma > \eta$ by assumption, $\chi_i \eta < 1$ must hold. But the solution of the last equation has the same structure as (37), $x_{it}$ hits zero in finite time.

Harvest per vessel, $h_i = \nu_i x_{i}^{\chi_i} e_i^{\epsilon_i}$, goes to zero with $x_i \to 0$ because each vessel’s profit-maximizing effort level is independent of the stock size, $e_i = (c_i \nu_i \epsilon_i / \omega)^{1/(1-\epsilon_i)}$ (this can be verified by plugging 11 into 4). The mass of vessels fishing species $i$, which can be calculated as total harvest divided by harvest per vessel, also goes to zero.

$$\lim_{x_i \to 0} m_i \leq \lim_{x_i \to 0} \frac{\Omega c_i^{-\sigma} x_i^{\sigma \chi_i}}{h_i} = \frac{\Omega c_i^{-\sigma}}{\nu_i e_i^{\epsilon_i}} \lim_{x_i \to 0} x_i^{(\sigma-1) \chi_i} = 0.$$ 

A.7 Proof of Proposition 3

Assume that species $i$ is collapsing at time $T_i$, and let $n_p$ be the number of stocks $j$ with strictly positive stock sizes at and shortly before $T_i$. Then differentiating
(12) for species $j \neq i$ with respect to time yields (with $x_j > 0$)

$$\frac{dH_j}{dt} = \gamma^\eta c_j^{-\sigma} x_j^{\chi_i} \sigma^{-1} \dot{x}_j \left[ \sum_{j=1}^{n} \left[ c_j x_j^{-\chi_i} \right]^{1-\sigma} \right]^{-\frac{\sigma-\eta}{\sigma-1}}$$

$$- \left[ c_j x_j^{-\chi_i} \right]^{-\sigma} (\sigma - \eta) \left[ \sum_{j=1}^{n} \left[ c_j x_j^{-\chi_i} \right]^{1-\sigma} \right]^{-\frac{\sigma-\eta}{\sigma-1}} \sum_{k \neq i} \chi_k c_k^{1-\sigma} x_k^{\chi_k (\sigma-1) - 1} \dot{x}_k$$

$$- \left[ c_j x_j^{-\chi_i} \right]^{-\sigma} (\sigma - \eta) \left[ \sum_{j=1}^{n} \left[ c_j x_j^{-\chi_i} \right]^{1-\sigma} \right]^{-\frac{\sigma-\eta}{\sigma-1}} \chi_i c_i^{1-\sigma} x_i^{\chi_i (\sigma-1) - 1} \dot{x}_i$$  (38)

For $t \to T_i$, the very last two factors become

$$\lim_{t \to T_i} x_i^{\chi_i (\sigma-1) - 1} \dot{x}_i \leq \lim_{t \to T_i} x_i^{\chi_i (\sigma-1) - 1} (\rho_i x_i - \Omega x_i^{\chi_i \sigma})$$  (39)

$$= \lim_{t \to T_i} \left( \rho_i x_i^{\chi_i (\sigma-1) - 1} - \Omega x_i^{2 \chi_i \sigma - \chi_i - 1} \right)$$  (40)

(i) Under the condition $2 \chi_i \sigma - \chi_i - 1 < 0$, or, equivalently, $\chi_i \sigma < (1 + \chi_i)/2$, this expression diverges to $-\infty$. Thus, $\lim_{t \to T_i} \dot{H}_j = \infty$. (ii) Under the condition $\chi_i \sigma > (1 + \chi_i)/2$ all terms in equation (38) remain bounded.

A.8 Proof of Proposition 4

We make use of the result shown in appendix A.3 that harvest of species of the least resilient species $i$ monotonically increases when the elasticity of substitution decreases. This implies that, everything else equal, harvest of this species $i$ is maximal for a level of $\sigma = 1$, and equal to

$$\bar{H}_i = \frac{\gamma}{n c} x_i^{\chi_i}$$  (41)

The proof then continues as follows: we will first show that for $\sigma = 1$ the stock of the least resilient species 1 will collapse under open access fishing if condition (17) holds. Because harvest of species 1 continuously decreases with $\sigma$ (cf. appendix A.4), a value $\sigma_1 > 1$ must exist for which harvest of species 1 is still so large that the stock of species 1 will collapse. Once the stock of species 1 has collapsed, species 2 is the least resilient among the remaining $n-1$ species. A similar argument then shows that a value $\sigma_2$, with $1 < \sigma_2 \leq \sigma_1$, must exist under the conditions given in the proposition such that the stock of species 2 will collapse. Iterating this argument, we conclude that a value $\sigma = \sigma_n > 1$ must exist such that the stocks of species $i = 1, \ldots, n$ will collapse one after another.
Using (41) and (2) in (1), and employing Assumption 1, stock dynamics of species 1 is given by

\[ \dot{x}_1 = \rho_1 x_1 (1 - x_1) - \frac{\gamma}{n c} x_1^\chi \]  

Under condition (17), the expression on the RHS of equation (42) is negative for all \( x_1 \), as we show in the following. The maximum of the RHS of (42) is determined by the condition

\[ \rho_1 (1 - 2 \bar{x}_1) - \chi \frac{\gamma}{n c} \bar{x}_1^{\chi-1} = 0 \]  

\( \iff \)  

\[ \frac{\gamma}{n c} \bar{x}_1^\chi = \frac{\rho_1}{\chi} \bar{x}_1 (1 - 2 \bar{x}_1) \]  

From condition (17) we conclude that the solution \( \bar{x}_1 \) to equation (44) must fulfill \( \bar{x}_1 < (1 - \chi)/(2 - \chi) \) because for all \( \bar{x}_1 \geq (1 - \chi)/(2 - \chi) \) the LHS of (44) is larger than the RHS of (44). By condition (17) this holds for \( \bar{x}_1 = (1 - \chi)/(2 - \chi) \):

\[ \frac{\gamma}{n c} \left( \frac{1 - \chi}{2 - \chi} \right)^\chi \geq \frac{\rho_1}{\chi} \frac{1 - \chi}{2 - \chi} \left( 1 - 2 \frac{1 - \chi}{2 - \chi} \right) = \frac{\rho_1}{2 - \chi} \frac{1 - \chi}{2 - \chi} \]

Because the LHS (RHS) of (44) is monotonically increasing (decreasing) in \( \bar{x}_1 \), this also holds for all \( \bar{x}_1 > (1 - \chi)/(2 - \chi) \).

Using (44) in the RHS of (42), we obtain

\[ \rho_1 x_1 (1 - x_1) - \frac{\gamma}{n c} x_1^\chi < \rho_1 \bar{x}_1 (1 - \bar{x}_1) - \frac{\gamma}{n c} \bar{x}_1^\chi \]

\[ < \frac{\rho_1}{\chi} \bar{x}_1 [\chi (1 - \bar{x}_1) - (1 - 2 \bar{x}_1)] < 0, \]

as \( \bar{x}_1 < (1 - \chi)/(2 - \chi) \).

As harvest \( H_1 \) of species 1 is monotonically decreasing with \( \sigma \) and the stock of species 1 collapses for \( \sigma = 1 \), a \( \sigma_c > 1 \) (but sufficiently small) must exist for which the stock of species 1 still collapses.

### A.9 Setting up the Planner’s Optimization Problem

The social planner maximizes the present value of utility with respect to \( y, q_i, e_i, \) and \( m_i \). Using (7) in (6), the problem can be written as

\[ \max_{y,\{q_i,e_i,m_i\}} \int_{t=0}^{\infty} \left[ y + \frac{\eta}{\eta - 1} \left( \sum_{j=1}^{\sigma_c} q_j \right)^{\frac{\sigma_c - 1}{\eta - \sigma_c}} \right] \exp(-\delta t) \, dt \]  

(47)
subject to \( q_i = m_i \nu_i x_i^{\chi_i} e_i^{\epsilon_i} \), (1), and (10). Plugging in (10) and using the co-state variables \( \pi_i \) for the constraints \( q_i = m_i \nu_i x_i^{\chi_i} e_i^{\epsilon_i} \) and \( \mu_i \) for constraints (1), the current-value Hamiltonian reads

\[
H = \gamma \frac{\eta}{\eta - 1} \left[ \sum_{i=1}^{n} q_i \frac{\sigma - 1}{\sigma} \right]^{\frac{\sigma}{\sigma - 1}} + \omega \left( 1 - \sum_{i=1}^{n} m_i e_i \right) - \sum_{i=1}^{n} m_i \phi_i \\
+ \sum_{i=1}^{n} \pi_i [m_i \nu_i x_i^{\chi_i} e_i^{\epsilon_i} - q_i] + \sum_{i=1}^{n} \mu_i [g_i(x_i) - m_i \nu_i x_i^{\chi_i} e_i^{\epsilon_i}] \tag{48}
\]

The necessary conditions for optimal effort and mass of vessels are

\[
\frac{\partial H}{\partial e_i} = 0 \quad \Leftrightarrow \quad [\pi_i - \mu_i] \epsilon_i m_i \nu_i x_i^{\chi_i} e_i^{\epsilon_i - 1} = \lambda \omega m_i \tag{49}
\]

\[
\frac{\partial H}{\partial m_i} = 0 \quad \Leftrightarrow \quad [\pi_i - \mu_i] \nu_i x_i^{\chi_i} e_i^{\epsilon_i} = \lambda \omega e_i + \lambda \phi_i \tag{50}
\]

Both conditions hold for all species \( i \). Using (49) in (50), we derive the optimal effort levels \( e_i = \epsilon_i \phi_i / ((1 - \epsilon_i) \omega) \). Evidently, the effort per vessel in the open-access setting (equation 32) is equal to the optimal effort level. To determine the optimal transitional dynamics toward the steady state we reformulate the regulator’s optimization problem by considering total harvest \( H_i = m_i \nu_i x_i^{\chi_i} e_i^{\epsilon_i} \) as the decision variable. Using \( e_i = \epsilon_i \phi_i / ((1 - \epsilon_i) \omega) \), the cost of fishing species \( i \) is

\[
\omega m_i \epsilon_i + \phi_i m_i = \frac{\omega H_i}{\nu_i x_i^{\chi_i} e_i^{\epsilon_i - 1}} + \frac{\phi_i H_i}{\nu_i x_i^{\chi_i} e_i^{\epsilon_i}} = \left[ \frac{\omega}{\nu_i} \left[ \frac{\epsilon_i \phi_i}{(1 - \epsilon_i) \omega} \right] e_i^{\epsilon_i - 1} + \frac{\phi_i}{\nu_i} \left[ \frac{\epsilon_i \phi_i}{(1 - \epsilon_i) \omega} \right] e_i^{\chi_i} \right] \frac{H_i}{x_i^{\chi_i}} = [\epsilon_i + 1 - \epsilon_i] \frac{\omega^{\epsilon_i} \phi_i^{1-\epsilon_i}}{\nu_i (1 - \epsilon_i)^{1-\epsilon_i} e_i^{\epsilon_i}} \frac{H_i}{x_i^{\chi_i}} \tag{51}
\]

Substituting this and the market-clearing condition \( q_i = H_i \) into (47) and rearranging leads to the optimization problem (18).

### A.10 Conditions for First-best Optimal Management

The current-value Hamiltonian for the regulator’s optimization problem is

\[
H = \gamma \frac{\eta}{\eta - 1} \left[ \sum_{i=1}^{n} H_i^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}} + \omega - \sum_{i=1}^{n} c_i H_i x_i^{\chi_i} + \mu_i [g_i(x_{ii}) - H_i] \}
\]

35
The first-order conditions for the optimal management of the fishery read
\[
\frac{\partial H}{\partial H_i} = 0 \quad \Leftrightarrow \quad \gamma H_i^{-\frac{\sigma}{\sigma - 1}} \left[ \sum_{j=1}^n H_j^{-\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma-1}{\sigma}} = c_i x_i^{-\chi_i} + \mu_i \quad (52)
\]
\[
\frac{\partial H}{\partial x_i} = \delta \mu_i - \dot{\mu}_i \quad \Leftrightarrow \quad \chi_i c_i H_i x_i^{-\chi_i - 1} + \mu_i g_i'(x_i) = \delta \mu_i - \dot{\mu}_i \quad (53)
\]
where both conditions (52) and (53) hold for all species \( i \). As the objective function is strictly concave and as all growth functions are strictly concave, the first-order conditions are also sufficient for a maximum. From (19) and (52) we derive the total harvest of species \( i \) as a function of the shadow prices \( \mu_i \):
\[
H_i = \gamma^n \left( c_i x_i^{-\chi_i} + \mu_i \right)^{-\sigma} \left[ \sum_{j=1}^n \left( c_j x_j^{-\chi_j} + \mu_j \right)^{1-\sigma} \right]^{\frac{\sigma-1}{\sigma}} \quad (54)
\]
Note that (54) holds both in and off the steady state. In a steady state the shadow price for fish does not change, i.e. \( \dot{\mu}_i = 0 \). Using this in (53), we obtain the steady-state shadow price of the stock of species \( i \)
\[
\mu_i = c_i x_i^{-\chi_i} \frac{\chi_i g_i(x_i)}{x_i \left( \delta - g_i'(x_i) \right)} = c_i x_i^{-\chi_i} \frac{\chi_i \rho_i \left[ 1 - x_i / \kappa_i \right]}{\delta - \rho_i \left[ 1 - 2 x_i / \kappa_i \right]} \quad (55)
\]
The social cost of harvesting one unit of species \( i \), reflected by \( c_i x_i^{-\chi_i} + \mu_i \), equals the current marginal cost of harvest plus the present value of additional future harvesting cost due to the marginally smaller fish stock.

A.11 Proof of Lemma 3
Let \( \pi_i = c x_i^{\chi_i - 1} \left[ 1 + \frac{\rho_i \left( 1 - \frac{x_i^*}{\kappa_i} \right)}{\delta - \rho_i \left( 1 - 2 \frac{x_i^*}{\kappa_i} \right)} \right] \). Given Assumption 1, total harvest \( H_i^* \) of species \( i \) is (by equation 12)
\[
H_i^* = \gamma \frac{\pi_i^{\sigma}}{\sum_{j=1}^n \pi_j^{1-\sigma}}
\]
Since \( d\pi_i / dx_i^* < 0 \), it is straightforward to show that \( H_i \) is monotonically increasing in \( x_i^* \), as \( x_i^* > \frac{\kappa_i}{2} \left( 1 - \frac{\delta}{\rho_i} \right) \). Hence, for the optimal steady state (see A.16) we obtain
\[
\frac{dH_i^*}{dx_i^*} > \rho_i \left[ 1 - 2 \frac{x_i^*}{\kappa} \right] \quad (56)
\]
Next we differentiate the equilibrium condition \( \rho_i x_i^* \left[ 1 - x_i^*/\kappa \right] = H_i^* \) with respect to \( \rho_i \). This yields

\[
x_i^* \left[ 1 - x_i^*/\kappa \right] = \left[-\rho_i \left[ 1 - 2 x_i^*/\kappa \right] + \frac{dH_i^*}{dx_i^*} \frac{dx_i^*}{d\rho_i} + \frac{\partial H_i^*}{\partial \rho_i} \right]
\]

By condition (56) the term in brackets on the RHS of this condition is positive. Also, it is straightforward to show that \( \partial H_i/\partial \rho_i < 0 \). We thus have \( dx_i^{oa}/d\rho_i > 0 \), i.e. the larger the intrinsic growth rate, the larger the first-best steady-state stock.

**A.12 Proof of Proposition 6**

The proof is analogous to the one of Proposition 1. Differentiating (20) with respect to the \( \sigma \) leads to

\[
\frac{\partial H_i}{\partial \sigma} = H_i \left[ -\ln \pi_i + \frac{\sum_{j=1}^{n} \pi_j^{1-\sigma} \ln \pi_j}{\sum_{j=1}^{n} \pi_j^{1-\sigma}} \right]
\]

(57)

Again the last term in brackets is independent of the species under consideration. Note also as in Lemma 1 that

\[
\frac{\sum_{j=1}^{n} \pi_j^{1-\sigma} \ln \pi_j}{\sum_{j=1}^{n} \pi_j^{1-\sigma}} = \ln \left[ \prod_{j=1}^{n} \pi_j^{\frac{1-\sigma}{\kappa}} \right]
\]

As reasoned in the proof of Lemma 1, the RHS of (57) must be negative for some species. Let \( j \) denote the species with the smallest shadow price that is larger than the weighted geometric mean. For all species \( i \) with \( i \leq j \) the RHS of equation (57) is still negative. Similar to the proof of Lemma 3, one can show \( \pi_1 < \pi_2 < \ldots < \pi_n \).

**A.13 Proof of Lemma 4**

With a similar argument as in the proof of Lemma 2 (A.5) we can show that if \( \chi_i \sigma > 1 \) then the optimal steady-state harvest of species \( i \) is smaller than natural growth for small stock sizes of species \( i \). This is because the optimal shadow price of species \( i \) is proportional to the open-access price \( c_i x_i^{-\chi_i} \). It remains to be shown that \( \rho_i > \delta \) is a sufficient condition for the steady-state stock to be strictly positive. Consider the steady-state shadow price of the stock \( i \) (equation 55). If \( \rho_i > \delta \), this
shadow price will diverge to \(+\infty\) as the stock approaches \(x_i = \frac{\eta_i}{2} \left(1 - \frac{\delta_i}{\rho_i}\right)\) from above. Hence the optimal steady-state stock must be greater than \(x_i\) with \(x_i > 0\) whenever \(\rho_i > \delta\).

### A.14 Proof of Proposition 7

Let \(n_1^*\) be the number of species for which \(\rho_i > \delta\), but \(\chi_i \sigma \leq 1\), while \(n_{\text{min}}^{\text{oa}}\) is the number of species for which \(\chi_i \sigma > 1\). By construction of \(n_1^*\), we obtain \(n_{\text{min}}^* = n_1^* + n_{\text{min}}^{\text{oa}}\). Thus we have \(n_{\text{min}}^* - n_{\text{min}}^{\text{oa}} = n_1^* \geq 0\) and hence \((n_{\text{min}}^* - n_{\text{min}}^{\text{oa}}) = n_1^*\), which is weakly decreasing with \(\sigma\).

### A.15 Conditions for Second-best Management

The current-value Hamiltonian for the regulator’s optimization problem is

\[
\mathcal{H} = \gamma \frac{\eta}{\eta - 1} \left[ H_k^{\frac{\sigma-1}{\sigma}} + \sum_{j \neq k} H_j^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\eta-1}{\eta}} \frac{\sigma}{\sigma-1} - \frac{2}{\sigma-1} \sum_{i \neq k} \lambda_i \left[ c_i x_i^{-\chi_i} - \gamma H_i^{\frac{1}{\sigma}} \left( H_k^{\frac{\sigma-1}{\sigma}} + \sum_{j \neq k} H_j^{\frac{\sigma-1}{\sigma}} \right) \right]^{\frac{\eta-1}{\eta}} \frac{\sigma}{\sigma-1} - \frac{2}{\sigma-1} \sum_{i \neq k} \lambda_i H_i^{-\frac{1}{\sigma}} \tag{58}
\]

The first-order conditions for the second-best management of species \(k\)

\[
\gamma H_k^{-\frac{1}{\sigma}} \left[ \sum_{j=1}^{n} H_j^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\eta-1}{\eta}} \frac{\sigma}{\sigma-1} - \frac{2}{\sigma-1} \sum_{i \neq k} \lambda_i H_i^{-\frac{1}{\sigma}} = c_k x_k^{-\chi_k} + \mu_k - \left( \frac{1}{\eta} - \frac{1}{\sigma} \right) H_k^{\frac{1}{\sigma}} \gamma \left[ \sum_{j=1}^{n} H_j^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\eta-1}{\eta}} \frac{\sigma}{\sigma-1} - \frac{2}{\sigma-1} \sum_{i \neq k} \lambda_i H_i^{-\frac{1}{\sigma}} \tag{58}
\]

\[
\chi_k c_k H_k x_k^{-\chi_k} + \mu_k g_k^0(x_k) = \delta \mu_k - \mu_k \tag{59}
\]

Conditions (58) and (59) must hold for the managed species \(k\). In addition, for the non-managed species \(i \neq k\) the following first-order conditions with respect to
$H_i$ and $x_i$ must hold for all species $i \neq k^{11}$

$$\gamma H_i^{-\frac{1}{\sigma}} \left[ \sum_{j=1}^{n} H_j^{\frac{\sigma-1}{\sigma}} \right]^{\frac{n-1}{\sigma-1}}$$

$$= c_i x_i^{-\chi_i} + \mu_i - \lambda_i \frac{\gamma}{\sigma} H_i^{-\frac{1}{\sigma}} \left[ \sum_{j=1}^{n} H_j^{\frac{\sigma-1}{\sigma}} \right]^{\frac{n-1}{\sigma-1}}$$

$$- \left( \frac{1}{\eta} - \frac{1}{\sigma} \right) \gamma H_i^{-\frac{1}{\sigma}} \left[ \sum_{j=1}^{n} H_j^{\frac{\sigma-1}{\sigma}} \right]^{\frac{n-1}{\sigma-1}} \sum_{i \neq k} \lambda_i H_i^{-\frac{1}{\sigma}}$$

$$\delta \mu_i - \dot{\mu}_i = \chi_i c_i H_i x_i^{-\chi_i} + \mu_i g'_i(x_i) - \lambda_i c_i x_i^{-\chi_i}$$

Since in a steady state we have $\dot{\mu}_i = 0$ for all species, we obtain

$$\mu_k = c_k x_k^{-\chi_k} \frac{\chi_k g_k(x_k)}{x_k \left[ \delta - g'_k(x_k) \right]}$$

(61)

for the managed species, just as in the first-best, and

$$\mu_i = c_i x_i^{-\chi_i} \frac{\chi_i (g_i(x_i) - \lambda_i)}{x_i \left[ \delta - g'_i(x_i) \right]}$$

(62)

for the non-managed species. Plugging this into (60), we find

$$c_i x_i^{-\chi_i} \frac{\chi_i (g_i(x_i) - \lambda_i)}{x_i \left[ \delta - g'_i(x_i) \right]} = \lambda_i c_i x_i^{-\chi_i} \frac{1}{\sigma H_i} + \sum_{i \neq k} \lambda_i \left( \frac{1}{\eta} - \frac{1}{\sigma} \right) H_i^{-\frac{1}{\sigma}} c_i x_i^{-\chi_i}$$

(63)

A.16 Proof of Proposition 8

The condition $\rho > \delta$ guarantees that the steady state stock sizes of the managed stocks are positive in the social optimum and in second-best. For identical species condition (54) for first-best steady-state management becomes

$$\gamma \left[ c x^* \right] \left[ 1 + \frac{\chi g(x^*)}{x^* \left[ \delta - g'(x^*) \right]} \right]^{-1} n^{-1} = g(x^*)$$

(64)

---

\(^{11}\) As stated in the main text, we assume an interior optimum here. In general we cannot exclude a priori that one of the non-managed stocks will collapse under second-best management (in particular if this stock already is about to collapse in the initial state). In such a case the set of conditions that apply for the non-managed species will be different as the number of stocks would change over time.
The LHS of this equation is monotonically increasing with \( x^* \), as

\[
\frac{d}{dx^*} \left\{ cx^* - \chi \left[ 1 + \frac{\chi g(x^*)}{x^* [\delta - g'(x^*)]} \right] \right\} = -\chi c x^{* - x - 1} \left[ 1 + \frac{\chi g(x^*)}{x^* [\delta - g'(x^*)]} \right] - c x^{* - x} \frac{\chi \delta}{[\delta - g'(x^*)]^2} < 0 \quad (65)
\]

For identical species condition (58) for second-best becomes

\[
\gamma g(x_k)^{-\frac{1}{2}} \left[ g(x_k)^{\frac{\sigma - 1}{\sigma}} + (n - 1) g(x_o)^{\frac{\sigma - 1}{\sigma}} \right]^{-1} = c x_k^{-\chi} \left[ 1 + \frac{\chi g(x_k)}{x_k [\delta - g'(x_k)]} \right] - \frac{\sigma - 1}{\sigma} \gamma g(x_k)^{-\frac{1}{2}} \left[ g(x_k)^{\frac{\sigma - 1}{\sigma}} + (n - 1) g(x_o)^{\frac{\sigma - 1}{\sigma}} \right]^{-2} (n - 1) \lambda g(x_o)^{-\frac{1}{2}} \quad (66)
\]

where \( x_k \) is the stock of the managed species and \( x_o \) are the stocks of the non-managed species that are fished under open access conditions. By symmetry, all the stocks sizes are the same. Rearranging, we obtain

\[
\left[ 1 + (n - 1) \lambda \frac{\sigma - 1}{\sigma} \frac{c x_o^{-\chi}}{\gamma} \right] \times \frac{n g(x_k)^{\frac{\sigma - 1}{\sigma}}}{g(x_k)^{\frac{\sigma - 1}{\sigma}} + (n - 1) g(x_o)^{\frac{\sigma - 1}{\sigma}}} \times \\
\gamma \left[ c x_k^{-\chi} \left[ 1 + \frac{\chi g(x_k)}{x_k [\delta - g'(x_k)]} \right] \right]^{-1} = g(x_k) \quad (67)
\]

The RHS of this condition is formally identical to the RHS of (64). The last factor on the LHS of (67) is identical to the LHS of (64). We proceed by showing that for \( x_k = x^* \) the first two factors on the LHS of (67) are both larger than one. As the LHS of (64) is monotonically increasing with \( x^* \), the solution \( x_k^{ab} \) of (67) must thus be smaller than \( x^* \).

To show that the first factor on the LHS of (67) is larger than one, we have to show that \( \lambda > 0 \). With identical species and \( \eta = 1 \), condition (63) becomes

\[
\frac{\chi g(x_o) - \lambda}{x_o [\delta - g'(x_o)]} = \frac{\lambda}{\sigma g(x_o)} + (n - 1) \lambda \frac{\sigma - 1}{\sigma} g(x_o)^{-\frac{1}{2}} \frac{g(x_k)^{\frac{\sigma - 1}{\sigma}} + (n - 1) g(x_o)^{\frac{\sigma - 1}{\sigma}}}{<1 (\text{concavity of } g(x))} < 1 (\sigma > 1)
\]

\[
\Leftrightarrow \lambda \left[ 1 - \frac{x_o [g'(x_o) - \delta]}{g(x_o)} \right] \frac{1}{\sigma} g(x_k)^{\frac{\sigma - 1}{\sigma}} + (n - 1) \frac{g(x_o)^{\frac{\sigma - 1}{\sigma}}}{g(x_k)^{\frac{\sigma - 1}{\sigma}} + (n - 1) g(x_o)^{\frac{\sigma - 1}{\sigma}}} < 1 (\sigma > 1)
\]

This shows that the term in curly brackets on the LHS is positive. As the RHS is also positive, we conclude that \( \lambda > 0 \).
The second factor on the LHS of (67) is also larger than one for \( x_k = x^* \), as the following argument shows: For identical species, the open-access constraint (24) for the non-managed species under second-best management becomes

\[
\gamma \left[ c x_o^{-\chi} \right]^{-1} \frac{g(x_o)^{\frac{-1}{\sigma}}}{g(x_k)^{\frac{-1}{\sigma}} + (n-1) g(x_o)^{\frac{-1}{\sigma}}} = g(x_o)
\]  

(68)

Comparing this condition with the open-access condition for symmetric species,

\[
\gamma \left[ c x_{oa}^{-\chi} \right]^{-1} n^{-1} = g(x_{oa})
\]  

(69)

we conclude that \( x_o > x_{oa} \) for \( g(x_k) > g(x_{oa}) \). As by assumption \( g(x^*) = g(x_{oa}) \), we conclude for \( x_k = x^* \) that

\[
\frac{n g(x_k)^{\frac{-1}{\sigma}}}{g(x_k)^{\frac{-1}{\sigma}} + (n-1) g(x_o)^{\frac{-1}{\sigma}}} > \frac{g(x_k)^{\frac{-1}{\sigma}} + (n-1) g(x_o)^{\frac{-1}{\sigma}}}{g(x_k)^{\frac{-1}{\sigma}} + (n-1) g(x_o)^{\frac{-1}{\sigma}}} = 1.
\]  

(70)

Thus, the second factor on the LHS of (67) is larger than one for \( x_k = x^* \). It also follows that \( x_o > x_{oa} \) and obviously \( x_o < x_k < x^* \). This concludes the proof of the proposition.

The socially optimal steady-state stocks are smaller than \( x_{MSY} = 1/2 \) if at \( x_{MSY} \) the LHS of (64) is larger than the RHS, i.e. if

\[
\gamma \left[ c \left( \frac{1}{2} \right)^{-\chi} \left[ 1 + \frac{\chi \rho}{2 \delta} \right] \right]^{-1} n^{-1} \geq \frac{\rho}{4}.
\]  

(71)

Rearranging leads to (26).

A.17 Conditions for Myopic Management

The current-value Hamiltonian for the regulator’s optimization problem is

\[
\mathcal{H} = \gamma \frac{\eta}{\eta - 1} \left[ H_k^{\sigma-1} + \sum_{j \neq k} H_j^{\sigma-1} \right]^{\frac{\sigma-1}{\sigma}} + \omega - \sum_{i=1}^{n} c_i H_i x_i^{-\chi_i} + \sum_{i=1}^{n} \mu_i \left[ g_i(x_{it}) - H_i \right]
\]

The first-order conditions for the optimal management of the fishery with respect to \( H_k \) and \( x_k \) read

\[
\gamma H_k^{\frac{1}{\sigma}} \left[ \sum_{j=1}^{n} H_j^{\sigma-1} \right]^{\frac{\sigma-1}{\sigma}} = c_k x_k^{-\chi_k} + \mu_k
\]  

(72)

\[
\chi_k c_k H_k x_k^{-\chi_k-1} + \mu_k g_k(x_k) = \delta \mu_k - \dot{\mu}_k
\]  

(73)

These conditions are formally identical with conditions (52) and (53) for the first-best optimal management, with the notable difference that they hold only for the managed species.
A.18 Proof of Proposition 9

Harvest of the managed species is

\[ H_{k}^{sb} = \gamma \left( \frac{\pi_{k}^{sb}}{\pi_{k}^{myopic}} \right)^{-\sigma} + \sum_{j \neq k} (c_{j} x_{j}^{-\chi})^{1-\sigma} \]

\[ H_{k}^{myopic} = \gamma \left( \frac{\pi_{k}^{myopic}}{\pi_{k}^{myopic}} \right)^{-\sigma} + \sum_{j \neq k} (c_{j} x_{j}^{-\chi})^{1-\sigma} \]

under second-best management

under myopic management,

where

\[ \pi_{k}^{sb} = c_{k} x_{k}^{-\chi} + \mu_{k}^{sb} - \left( \frac{1}{\eta} - \frac{1}{\sigma} \right) H_{k}^{1-\frac{1}{\sigma}} \gamma \left( \sum_{j=1}^{n} H_{j}^{1-\frac{1}{\sigma}} \right)^{-\frac{1}{\eta - 1}} \]

\[ \pi_{k}^{myopic} = c_{k} x_{k}^{-\chi} + \mu_{k}^{myopic} \]

Assume myopic management of species \( k \) is always the same as second-best management. Then we would have \( \mu_{k}^{myopic} = \mu_{k}^{sb} \) at any time (by equations 73 and 59). Thus, \( \pi_{k}^{myopic} > \pi_{k}^{sb} \), and \( H_{k}^{myopic} < H_{k}^{sb} \), which is a contradiction of the assumption that myopic management is the same as second-best management. A similar contradiction is derived if one assumes that myopic management is less strict, i.e. if harvest is higher under myopic management than second-best management. We thus conclude that myopic management is stricter than second-best management.

References


FAO (2010), The State of World Fisheries and Aquaculture - 2010 (SOFIA). FAO Fisheries and Aquaculture Department, Rome.


Skonhoft, A. (2009), The mixed bless of more modern fishing technology. University of Trondheim, mimeo.


