

Berichte
aus dem
Institut für Meereskunde
an der
Christian-Albrechts-Universität . Kiel
Nr. 30

DESIGN CONCEPTS FOR A SHALLOW
WATER VELOCITY PROFILER

AND

A DISCUSSION OF A PROFILER
BASED ON THE PRINCIPLES OF
GEOMAGNETIC INDUCTION

Technical Report

by

Thomas B. Sanford

Institut für Meereskunde
an der Universität Kiel

and

Woods Hole Oceanographic Institution
Woods Hole, Massachusetts

Kiel

1977
DOI 10.3289/IFM-BER-30

Abstract

This report discusses several techniques which have been or could be used to measure absolute velocity profiles in water less than 50 m deep. The method requirements are

1. Depth: 0-50 m
2. Accuracy: a) ± 1 cm/s in velocity
b) ± 20 cm in depth
3. Resolution: a) ± 0.5 cm/s in velocity
b) ± 5 cm in depth

After various alternatives are briefly discussed, a method based on geomagnetic induction is described in more detail. This method infers the absolute velocity profile from measurements of the electric currents and fields generated by the motion of the instrument and the surrounding sea water through the earth's magnetic field. The report presents the most significant concepts and theory involved in this approach.

Design Concepts for a Shallow Water Velocity Profiler

I. Introduction

The variations of ocean currents over distances from the whole water depth down to a few centimeters are of great interest to oceanographers. Currents not only transport heat, salt, sediments and pollutants but also cause these quantities to be mixed vertically and horizontally. In shallow water this mixing is vitally important in distributing energy, momentum, and the passive quantities. Moreover, the vertical structure of the currents determine the amount of kinetic energy which is dissipated in the water and near the sea bottom.

An adequate description of the vertical structure of ocean currents has been a goal of oceanographers for many years. The most frequently used method to measure the velocity profile is the moored array of current meters. By this means it is usually impossible to deploy a sufficient number of sensors in the vertical to describe the structure; these measurements suffer from spatial aliasing. Recent developments have partially overcome this problem by observing the horizontal velocity at very many levels as the instrument moved vertically in the water column.

Many of the existing techniques can be applied in shallow water (depth less than 100 m) with some success. However, the velocity structure in shallow water can be very complex, having strong mean currents, large vertical shear and turbulence. These characteristics require that the method be very rugged, easily deployed and recovered, and that it have high resolution in time, velocity and depth. On the other hand, shallow water depth permits the application of measurement schemes which would be considerably more difficult or inaccurate in deeper water.

The purposes of this report are first to establish a reasonable set of measurement specifications or requirements for a shallow water velocity profiler, to review a variety of possible measurement techniques and, finally, to describe one or two of the most promising methods in more detail.

At the outset of the study it is important to formulate a reasonable or

attainable level of profiler performance. Considering the state-of-the-art in current measurement, it is extremely difficult to achieve all requirements with one velocity sensor. That is, sensors which measure currents over the full water depth generally have a coarse vertical resolution, say 1-10 meters, and sensors which respond well to small vertical scale current variations are insensitive to larger vertical scale structure. Although there are exceptions to this rule, the exceptions involve other problems, such as requirements for power or the stability of the platform, which are difficult to overcome. Therefore, the approach followed has been to seek a method capable of measuring currents over distances of 1-50 meters. Then current fluctuations on small vertical scales will be observed by a second method used in conjunction with the first instrument.

Listed below are the system requirements which if attained would yield an absolute velocity profiler of considerable utility. The requirements are:

- a) Depth: Fullwater column, 0-50 m
- b) Accuracy: 1) ± 1 cm/s in velocity every 1 m
2) ± 20 cm in depth
- c) Resolution: 1) ± 0.5 cm/s in velocity every 1 m
2) ± 5 cm in depth

In addition to the above list of required performance, there are a number of desired features which the profiler should contain. The desired features are:

- a) Easy and rapid deployment from a small research vessel (weight less than 50 kg)
- b) Real-time data storage and display on the vessel
- c) Ability to obtain repeated profiles at the same location, possibly as autonomous operation
- d) Ability to obtain temperature and electrical conductivity profiles with the same instrument
- e) Useful in water much deeper than 50 m.

The various profile methods have been divided into three categories depending on the type of measurement being made. The \ddot{X} mode represents techniques which basically measure acceleration. Similarly, the \dot{X} and X modes represent methods which directly measure velocity and displacement, respectively.

Under each of the three categories one or more methods are described and their expected performances are discussed. The catalog of methods is comprehensive yet not complete. Undoubtedly, additional methods have been envisioned or even operated. Yet, the methods discussed represent a rather complete sample of the available approaches.

II. Velocity Profiler Techniques

A. The \ddot{X} or acceleration mode

1. Inertial guidance methods.

Accelerometers and attitude sensors are used to measure the acceleration experienced by a freely falling instrument probe. The accelerations are used, together with relationships governing the motion of the probe in the surrounding water, to compute a velocity profile. The method depends on sophisticated and costly inertial guidance instrumentation and on extensive knowledge of the motion of the probe in shear flow. Thus, the accuracy depends directly on the sensors and the knowledge of the relative motion of the falling probe. Due to drift in the acceleration sensors the probe should fall rather rapidly (~ 10 - 100 cm/s) and must remain absolutely stationary on the sea floor. Because it remains on the bottom, the constant of integration is known; that is, $V(\text{bottom}) = 0$. The advantage of the method are that it is self contained, and it profiles the whole water column. The disadvantages are that it is costly to build and maintain, extensive computations are required to yield a velocity profile and it must remain stationary on the sea floor.

2. Measurement of relative velocity

In these methods a measurement is made of the velocity difference between a freely-falling probe and the surrounding water. These methods are listed in this category because the relative velocity must be considered to be an effective acceleration applied to the probe. Thus, the velocity profile must be determined by an integration of the drag accelerations resulting from the measured relative velocities. Instruments such as these have been developed by Osborn (1974), Simpson (1972), Rossby (1974) and several others. The vertical resolution depends upon the type of sensors used but has been as small as several centimeters and as large as 10 m. The instrument must remain stationary on the sea floor so that the absolute velocity of the water there can be measured. No relative velocity profiles have been integrated to yield absolute profiles, at least none have been published. The advantages of the method are that it is self contained, operates over the whole water column and falls rapidly. The disadvantages are that it is a free probe, that the profiles require extensive computation

and probe calibration and that the instrument must come to rest on the sea floor.

B. The \dot{x} or velocity sensing mode

1. A current meter sliding on a vertical wire

In this method which has been pioneered by researchers at the Institut für Meereskunde in Kiel and the University of Miami, a current meter is constrained to move along a nearly vertical, taut wire. Two basic configurations of this approach exist: the wire is either moored to the sea bottom or attached to a drifting ship. A large amount of tension is applied to the wire by surface buoyancy or by a large weight on the end of the wire depending on where the wire is attached. The purpose of the taut wire is to provide a stable reference frame from which to make measurements of the current flowing past the wire. When the measurements are made from a drifting ship it is necessary to correct for the ship's motion in order to obtain absolute velocities. In the moored configuration, a vessel deploys a bottom anchored mooring. The current meter falls down along this wire until it reaches the anchor or some preset depth. Then the whole mooring plus current meter is recovered (Müller et al., 1974) or the current meter system changes its buoyancy and returns toward the surface (the cyclosonde system of Van Leer et al., 1974). In the case of the autonomous cyclosonde system, the profiler can be induced to traverse the moored line repeatedly for many cycles. When used from a ship, the method has two additional forms depending on whether or not the ship is anchored. From an anchored ship, a heavily weighted cable is lowered on which the profiler operates (Düing and Johnson, 1972). The weight on the end of this cable does not touch the bottom but rather establishes a taut wire reference frame from which to measure the velocity profile. In the case when the ship is not anchored, the method (Müller et al., 1974) is exactly the same, only the measured profile is relative to the motion of the ship. Different types of current meters may be used with this method but only ones small in size and light in weight are really suitable (e.g. Aanderaa model). The advantages of these methods are that they can use commercially available current meters, they work well in strong unidirectional (with respect to depth) currents and they can be operated for extended periods of time (>10 days). Some of the disadvantages are that the measurements are relative to a wire which

may be neither vertical nor motionless (even if anchored at one end), that it is difficult to prevent vertical motion of the wire from causing measurement errors and that these methods generally do not provide good measurements near the surface vessel nor near the sea floor.

2. Current meter raised or lowered by a winch

In this method a winch situated either on the sea floor or on the ship raises or lowers a current meter. This method differs in an important way from that just described. In this case a previously deployed taut wire is not used. Rather, the current meter is the final element on the wire and, thus, does not benefit from the more stable reference frame of the taut wire. For this reason, lowered current meter measurements from a drifting ship are generally not good. On the other hand, such measurements from a bottom mounted winch operated system can be of high quality. Assuming adequate velocity sensors and sea-bed winch are present, it is then only necessary to raise and lower the current meter slowly enough that accurate measurements of absolute velocity are obtained. If the sensors are moved too rapidly, then the lateral motion of the sensor will result in measurement errors. Researchers at the Institut für Meereskunde have developed several of these systems (Siedler and Krause, 1964). The advantages of the bottom mounted method are that a variety of sensors can be used, it can operate for extended periods of time, and it can provide real-time data display. The disadvantages are that it uses so much electrical power that it must either be operated directly from a research vessel, offshore platform or shore base, and that the deployment and recovery of such instrumentation is complex and weather dependent.

3. Velocity profiler operating on acoustic Doppler effects.

A moving acoustic sender and receiver will receive returns from acoustic scatters in the water and on the sea floor. The frequency of the returned signals will be changed in relation to the motion of the sender and receiver relative to the principal acoustic scatters. Thus, an instrument which emits an acoustic signal of known frequency will receive returns or echoes of different frequency. Using a conically shaped output beam of narrow width ($\sim 5^\circ$ or less) it is possible to measure the motion of an instrument over the sea floor in water less than 300 m deep. This method has been developed for purposes of ship navigation, and a free fall device has been

developed by Drever and Sanford (1976) at the Woods Hole Oceanographic Institution. The advantages of this method for profiling are that it is self contained and yields absolute profiles rapidly. The disadvantages are that it may be influenced by strong gradients to the sound speed structure and by strong, mid-depth acoustic scatters and that it does not operate well within about 10 m of the sea floor.

4. Electromagnetic Method

This method is based on the measurements of weak electric currents in the sea due to the motion of the sea water through the earth's magnetic field. Measurements of the electric currents in the water column and of the electric field near the sea floor permit the determination of the absolute velocity profile. The method consists of measurements of the potential differences between two horizontally spaced electrodes as the instrument falls or rises through the water column. The measurements by themselves yield a profile of relative currents in the manner of many profiles made by Sanford. However, if additional electric measurements are made on the sea floor while the instrument is stationary, then this additional information permits the relative profile to be converted into an absolute one. No instrument has yet been developed which measures both the electric current profile and the sea floor electric field. However, numerous deep-water electric current profiles have been measured by Sanford with a free fall instrument (Drever and Sanford, 1970; Sanford et al., 1974). Also, another instrument to measure the near bottom electric field is being developed at Woods Hole by Sanford and Drever. A similar instrument has been developed by Filloux (197). The advantages of this approach are that the method has high velocity resolution, is self contained, may be electrically connected to the ship via a wire spool (in the future, it may be possible to operate the profiler from the ship's winch or hydrographic wire and, yet, not suffer from the errors occurring with conventional current meters) for real-time data display and the method is very rapid and mobile. The disadvantages are that it must remain stationary for several minutes on the sea floor and that it may be sensitive to strong vertical gradients in temperature and salinity. Also there may be significant, near bottom electric fields due to mineral deposits and decaying organic matter.

C. X or displacement mode

The position versus time of a free probe is acoustically tracked by a hydrophone array. The array is preferably installed on the sea floor since installation of the array on a moving ship severely degrades the measurement. An array of 3 or more hydrophones rests on the sea floor. A pinger is released from the array frame and floats toward the sea surface. The tracking method is called spherical when 3 hydrophones are used to detect the time of arrival of an individual acoustic pulse. Hyperbolic tracking is possible when the exact time of pulse emission is known. By either method, the position of the free probe is determined relative to the frame. Additional measurements are needed to determine the orientation of the hydrophone array (e.g. compass heading and vertical tilt). For shallow water applications, it is preferable for the hydrophone array to be electrically and mechanically attached to the ship. The data storage, computations and display would be done aboard the ship. No known mobile system exist. Many velocity profiles have been taken over fixed hydrophone arrays (Rossby 1969, 1974) and over specially installed acoustic transponders (Pochapsky, 1976). The advantages are that absolute velocity profiles can be computed, more than one rising probe can be used so that repeated profiles are possible, real-time display is possible and that most of the electronics is aboard the ship. The disadvantages are that it may be difficult to deploy and recover in all sea states, the free probe may be costly if not recovered, the ship must carefully maneuver while the cable is attached to the array and that the method may be sensitive to strong gradients in the vertical profile of sound speed.

III. Discussion of EM Approach

A. Justifications for Choice of the EM method over the alternatives

Of the various methods presented in the previous section, the free-fall EM approach appears to have the most promise for meeting the requirements. Moreover, it represents a new approach to the measurement of shallow water flow and is capable of meeting most of the desirable capabilities. The EM approach is especially good in strong flows in which it is difficult to deploy moorings or maintain station with a vessel. Also in weak flows it has no mechanical

stall or threshold speed below which it no longer registers. The method presents the possibility of making deep water absolute profiler and for monitoring transport from the sea floor (Filloux, 1974). It should be mentioned that the method assumes that there are no significant changes in the horizontal electric field with respect to depth. In shallow water there may exist strong, near bottom variations to the electric field arising from electrochemical effects in the sediments. The existence and strength of such fields needs to be investigated.

B. The interpretation of the measurements

The EM approach is a departure from the more usual measurements of velocity by the direct displacements of bodies and rotors. For this reason the method is somewhat more difficult to understand and more attention must be given to the theory of operation.

Electric fields and electric currents are generated in the sea by the motion of the sea water through the earth's magnetic field. The basis of the EM method is the measurement and interpretation of these electrical effects. That is, the electrical measurements are interpreted in terms of the velocity which would produce the measured fields.

The electric fields ($-\nabla\phi$) and electric currents (\vec{J}) generated by movement through a magnetic field are related to the velocity field (\vec{v}) by Ohm's Law for a moving conducting medium:

$$(1) \quad \nabla\phi = \vec{v} \times \vec{F} - \vec{J}/\sigma$$

where

$$\begin{aligned} \vec{F} &= \text{geomagnetic field} \\ \sigma &= \text{electrical conductivity} \end{aligned}$$

The observed potential gradient is the combination of the source function ($\vec{v} \times \vec{F}$) and the electric current response. For a constant and uniform geomagnetic field, the source function varies in time and space according to

the local \vec{v} . Likewise, the electric current response will vary in space and time according not only to the local \vec{v} but also to the velocity field over a large region.

Ohm's Law can be rewritten as

$$(2) \quad \vec{v}_H = \frac{1}{F_z} \vec{k} \times (\nabla \Phi + \vec{J}/\sigma) + \frac{F_H}{F_z} v_z \vec{j}$$

where

- F_z = vertical component of the geomagnetic field
- F_H = horizontal component of the geomagnetic field
- $\vec{i}, \vec{j}, \vec{k}$ = unit vectors in the x/y and vertical directions (y positive toward geomagnetic north; z positive upward)
- \vec{v}_H = horizontal velocity (u,v)
- v_z = vertical velocity component (w)

According to the above equation, the horizontal velocity components (u,v) are

$$u = -\frac{1}{F_z} \left(\frac{\partial \Phi}{\partial y} + J_y/\sigma \right)$$

(3)

$$v = \frac{1}{F_z} \left(\frac{\partial \Phi}{\partial x} + J_x/\sigma \right) + \frac{F_H}{F_z} w$$

These are the most general and exact equations possible in the mathematical description of motional induction. They are valid regardless of the characteristics of the flow, bottom topography or water and sediment conductances. However, the measurement of $\nabla \Phi$ is difficult to obtain since it must be measured by stationary electrodes. On the other hand, measurements of \vec{J}/σ can be made readily as the voltage between horizontally-spaced electrodes mounted on a falling probe which is free to move horizontally with the flow. Even if both

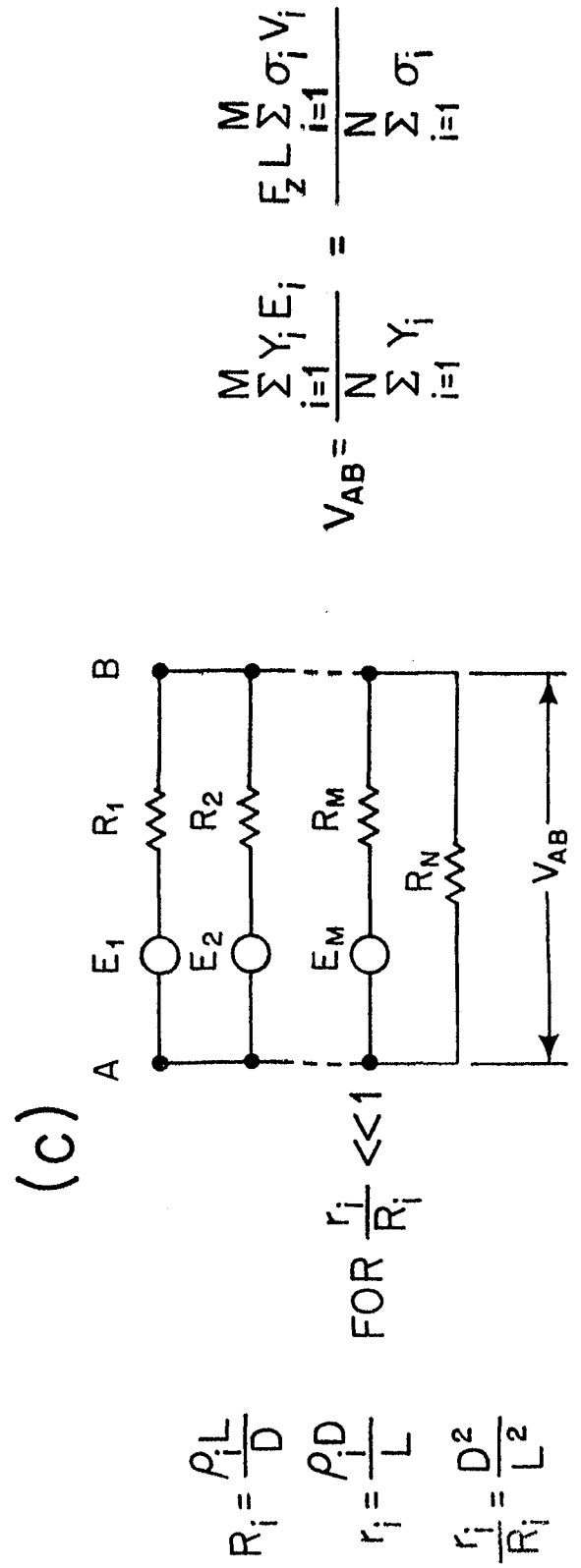
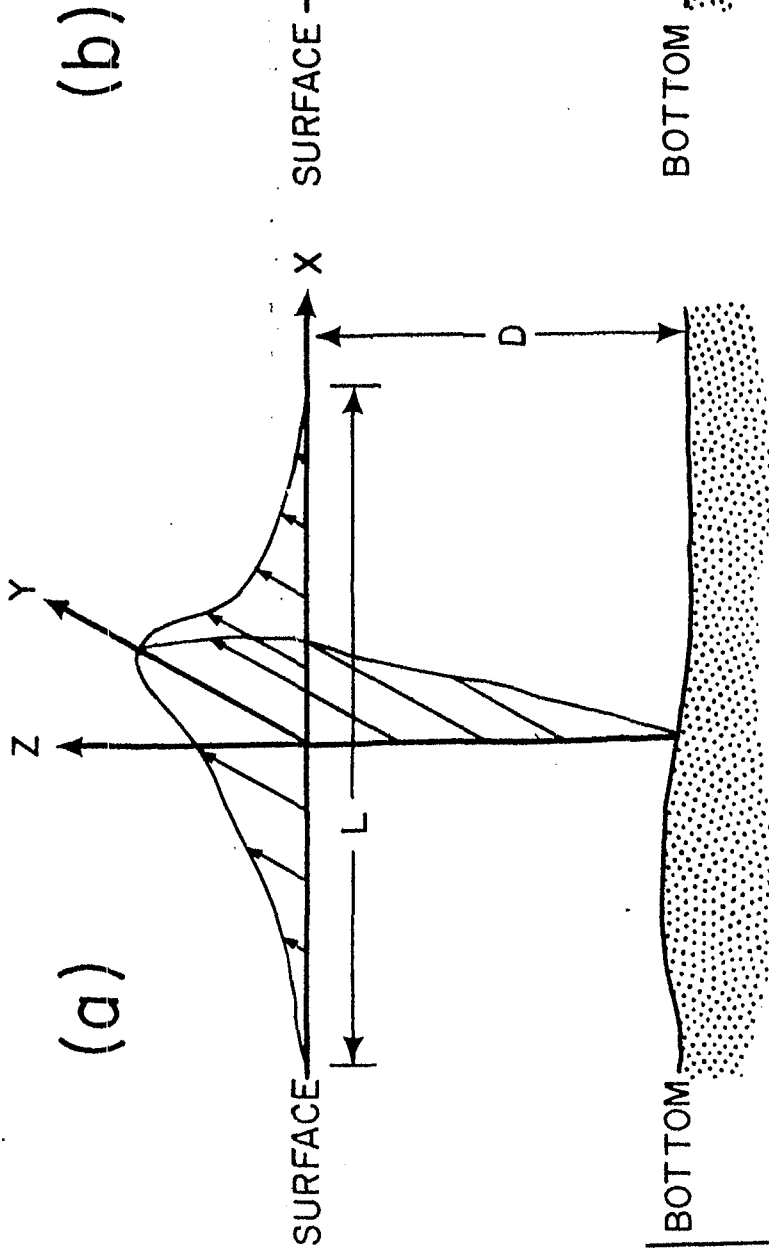
$\nabla\Phi$ and J/σ could be measured, the velocity component, v , is still in error by the term proportional to w .

The difficulty in making $\nabla\Phi$ measurements is reduced greatly in situations in which $\nabla\Phi$ does not vary with depth. Such situations arise within flows which are broad compared with the water depth; that is, within flows which do not change significantly over a horizontal distance comparable to the water depth or to its own vertical variations. In these cases the potential gradient is essentially independent of depth and, hence, is measurable on the sea floor.

A simple model of induction within ocean currents should help establish physical insight into the role played by the ratio of water depth to current width in motional induction. Consider a two-dimensional ocean current as in Fig. 1 a, having variations in horizontal velocity (the vertical velocity component is zero) in the cross-stream and vertical coordinates. Due to motion through the vertical component of the geomagnetic field, EMF's will arise everywhere within the current. The electric induction and resulting electric currents can be modelled by an electric network as shown in Fig. 1 b consisting of voltage sources, E_i , and resistances, R_i . In this model we consider the ocean current to be composed of M layers insulated from each other except at the ends. The parameter, D/L , largely determines the ratio of vertical to horizontal resistances. It is almost always true that $D/L < 1$ in oceanic flows, a condition which permits r_i to be ignored as in Fig. 1 c and the network to be easily solved.

The induced electric field is independent of depth and proportional to a conductivity-weighted average velocity. The conductivity of the water and the sediments are both important. Let $\partial\Phi/\partial x = F_z \bar{V}^*$, where the conductivity weighted average, \bar{V}^* , is generally little different from \bar{V} except where the sea is shallow or the bottom is unusually conductive.

The electric current in each R_i is then proportional to $F_z (V_i - \bar{V}^*)\sigma_i$. In a continuous medium the electric current density is $J_x = F_z (V(z) - \bar{V}^*)\sigma(z)$. These electric currents circulate in a vertical plane perpendicular to the flow. As previously mentioned, the expression for the electric current is of interest because potential measurements between electrodes moving with the



local fluid velocity will measure only the voltage drop arising from the electric currents. It can be easily seen that these expressions for $\partial\Phi/\partial x$ and J_x/σ satisfy equation 3 to within an error of order w .

More general analytic expressions were derived by Sanford (1971) giving explicit relationships between the electric current density and the velocity field. For the present purposes it is helpful to evaluate these expressions for an ocean current which is broad compared with its depth. The dominant contributions to \tilde{J} are

$$(4) \quad \begin{aligned} \tilde{J} &= \nabla \times \int_{-H}^0 \tilde{k} \times [(\tilde{v} - \bar{v}) \times \tilde{F}] d\xi + \sigma \overline{(\tilde{J}/\sigma)} \\ &- \nabla_H \int_{-H}^0 \sigma \tilde{k} \cdot \tilde{v} \times \tilde{F} d\xi \end{aligned}$$

where

$$\begin{aligned} \overline{(\quad)} &= \frac{1}{H} \int_{-H}^0 (\quad) d\xi = \text{vertical average of } (\quad) \\ \nabla_H &= \partial/\partial x \, \tilde{i} + \partial/\partial y \, \tilde{j} \end{aligned}$$

Equation 4 differs only from Equation 20 of Sanford (1971) in that all depth-independent terms, except the \bar{v} term, to order H^2/L^2 are combined into the expression $(\overline{\tilde{J}/\sigma})$.

If we evaluate Equation 4 in terms of its vector components, we obtain

$$(5) \quad \begin{aligned} J_x/\sigma &= F_z (v - \bar{v}) - F_H (w - \bar{w}) + \overline{(J_x/\sigma)} - \frac{\partial}{\partial x} \int_{-H}^z F_H u d\xi \\ J_y/\sigma &= -F_z (u - \bar{u}) + \overline{(J_y/\sigma)} - \frac{\partial}{\partial y} \int_{-H}^z F_H u d\xi \end{aligned}$$

where

$$\tilde{F} = 0, F_H, F_z = \text{horizontal and vertical components of the geomagnetic field}$$

According to Equations 1 and 5 the potential gradient at any level is

$$\begin{aligned} \partial\Phi(z)/\partial x &= \partial\Phi(-H)/\partial x + \partial/\partial x \int_{-H}^z F_H u d\xi \\ (6) \quad \partial\Phi(z)/\partial y &= \partial\Phi(-H)/\partial y + \frac{\partial}{\partial y} \int_{-H}^z F_H u d\xi \end{aligned}$$

Thus, except for the last terms in Equation 6, the potential gradient is independent of depth. Since it is difficult to measure $\nabla\Phi$ at sea except from the sea floor, the error in such measurements will be of order H/L , which is a generally small quantity. If we substitute $\nabla\Phi(-H)$ and $J(z)/\sigma(z)$ into Equation 2 and denote the left hand side as \tilde{v}' (since it is no longer exactly equal to \tilde{v} , the true velocity) we obtain:

$$\begin{aligned} \frac{-\tilde{j} \cdot (\nabla\Phi(-H) + J/\sigma)}{F_z} &\equiv u'(z) = u(z) + \frac{F_H}{F_z} \frac{\partial}{\partial y} \int_{-H}^z u d\xi \\ (7) \quad \frac{\tilde{i} \cdot (\nabla\Phi(-H) + J/\sigma)}{F_z} &\equiv v'(z) = v(z) - \frac{F_H}{F_z} \left(\frac{\partial}{\partial x} \int_{-H}^z u d\xi + w(z) \right) \end{aligned}$$

These relations can be slightly rewritten. Assume that $\nabla \cdot \tilde{v} = 0$ and $w(-H) = -\nabla_H H \cdot \tilde{v}(-H)$ (i.e. no velocity component normal to the sea floor):

$$\begin{aligned} u'(z) &= u(z) + \frac{F_H}{F_z} \int_{-H}^z \frac{\partial u}{\partial y} d\xi \\ (8) \quad v'(z) &= v(z) + \frac{F_H}{F_z} \int_{-H}^z \frac{\partial v}{\partial y} d\xi \end{aligned}$$

Thus combination of electric measurements while a probe is freely falling (J/σ) and while it is on the bottom ($\nabla\Phi(-H)$) yield an absolute velocity profile to an accuracy limited by the horizontal shear of the current which due to continuity is also about equal to the vertical velocity component. Therefore the error inherent in this method is about equal to w .

C. Concept of an instrument and discussion of design considerations

The concept of the method will be presented here in more detail. The proposed instrument will measure the potential differences between several horizontally spaced points around an instrument housing. These measurements will be made both as the instrument falls toward and rises from the sea floor and while it remains stationary on the bottom.

In the design of such an instrument there is one very important practical consideration. This is that available electrodes are not sufficiently stable to measure the voltage between two points to within an accuracy of 1 microvolt. Actually, the self EMF or offset of electrodes can drift over a span of millivolts. Hence, it is necessary to separate the offset voltage of the electrodes from the potential difference due to the motional induction. The extent to which the offset and desired signals can be separated will largely determine the resolution of the method.

Presently, only one method is known to achieve the required performance: the exchange of the electrodes. This exchange of electrodes can be achieved either by physically interchanging the electrodes or by using tubing and valves. In the first instance, the positions of the electrodes are exchanged by allowing the instrument to rotate as it falls. This approach is used by Sanford et al. (1974) in their EMVP. This method works well and is simple to implement. However, the positions of the electrodes are never exactly interchanged since the instrument falls about 2-3 meters in the time required for it to make one-half a rotation. For this reason, the EMVP has a vertical resolution of about 5 m. Clearly, a faster rotation rate could be used especially in shallow water where the speed of descent and rise need not be so rapid as in deep water.

The second solution to the offset problem is to use sea water filled tubes and valves. In this method electrode A is attached to the end of tube 1 and electrode B is attached to the end of tube 2. The voltage between electrodes A and B consists of the offset plus the voltage between the other ends of the two tubes. Then by valves the tubes and electrode pairs are exchanged: that is, electrode A is connected to tube 2 and B is connected to tube 1. Now the voltage between electrodes A and B consists of the same offset minus the

voltage between the ends of the tubes. The offset and desired potential difference can be separately determined since we have two independent equations for two unknowns. It should be noted that in this method, as in the previous one, the positions of the ends of the tube may be changing as the probe moves. Thus, a rapid electrode exchange is needed when the currents change strongly with position especially with depth.

Although the electrode exchange should be performed quickly, it may not have to be done frequently. The frequency with which the electrodes must be exchanged depends primarily on how rapidly the offset changes. If the offset drifts only slowly, then the electrodes need not be exchanged frequently. In fact if they did not change at all during the period of a deployment, then the offset could be determined before and after use. Such electrode stability is probably not achievable but some effort should be given to an evaluation of electrode performance from this point of view.

Assuming that the elimination of the electrode offset is achieved, there are a few further practical considerations.

1. Regardless of the method used to determine the electrode offset, the electrodes must be shielded from temperature and salinity fluctuations. If the sea water surrounding one electrode changes by 1°C in temperature or 1‰ in salinity the electrode potential difference changes by 350 μv or 500 μv , respectively. This effect is very significant since the temperature and salinity generally change by much more than one unit during a profile in any sea. Careful design of the electrode system will prevent such signals from appearing at the frequency of the desired signal.
2. Since measurements of electric currents in the sea are being made, it is clearly important that the instrument itself not produce electric currents. Normally, such stray electric currents arise by electrolytic action between dissimilar metals in the presence of sea water. This problem is not as severe when the instrument and electrode arms rotate together since the electric currents also rotate and are observed as a DC bias. In the case of a valve-type instrument, the corrosion generated currents will produce a bias which is not separable from the desired signal. The construction of the instrument must not contain these electrochemical cells.

3. The rate of fall of the instrument through the horizontal component of the earth's magnetic field produces a large signal. In order to eliminate the fall induced signal, in later processing, good measurements of pressure are required from which the rate of change of pressure or fall speed is computed. This requirement should not be difficult to achieve in shallow water.

A discussion previously was presented of the electric fields and currents generated by water motion through the earth's field. Now it is vital to compute the signals actually measured by the proposed instrument.

The electric field sensed by the instrument depends on the applied electric currents and on the motion of the vehicle relative to the local water. The electric current pattern is modified in the vicinity of the instrument. Because of the insulating surface of the vehicle, the horizontal electric currents must diverge to pass by rather than to continue through the skin. The requirement that the normal component of electric current density vanish at the insulating skin is satisfied by the establishment of a surface charge distribution. It is necessary to account for the influence of this charge distribution on the measured potential differences.

The additional induction locally generated by relative motion must also be considered. Since it is reasonable that the vehicle is not always moving exactly with the local horizontal velocity, a question arises as to which velocity is measured; that of the local water or that of the vehicle. Moreover, it is proposed that the instrument become stationary on the sea floor so there may be a large flow past it at this time.

In order to compute the electrical response of the instrument, it is necessary to know its geometric shape. Based on the concept of the device and previous experience, the instrument will probably be similar to a truncated cylinder to which various devices are attached. Unfortunately, the analytical solutions for the response of such a body are very complex. Therefore, it is suggested that its form be similar to a prolate spheroid, a form which is analytically more simple.

The solutions are found in terms of spheroidal harmonics about the body

spheroid having foci at $x = y = 0$ and $z = \pm a/2$. It is convenient to align the x, y, z coordinate axes with the geomagnetic elements. Then the x axis points toward geomagnetic east; the y axis points toward geomagnetic north; the z axis points vertically up.

The coordinate transformations are

$$\begin{aligned} x &= \frac{a}{2} \sqrt{(\xi^2 - 1)(1 - \eta^2)} \cos \phi \\ (9) \quad y &= \frac{a}{2} \sqrt{(\xi^2 - 1)(1 - \eta^2)} \sin \phi \\ z &= \frac{a}{2} \xi \eta \end{aligned}$$

$$\xi = (r_1 + r_2)/a ; \quad \eta = (r_1 - r_2)/a ; \quad \phi = \tan^{-1} y/x$$

where

$$\begin{aligned} r_1^2 &= x^2 + y^2 + (z + a/2)^2 \\ r_2^2 &= x^2 + y^2 + (z - a/2)^2 \end{aligned}$$

Figure 2 presents a prolate spheroidal body and the orthogonal coordinate system external to the body which is taken to be the surface $\xi = 1.02$. Assume measurements are made of the potential differences between the ends of L_1 and of L_2 .

The general solution to the response of such a body to relative motion and applied electric currents (but the last term in Equation 4 is ignored) is presented by Sanford et al. (1974, pp. 43-53).

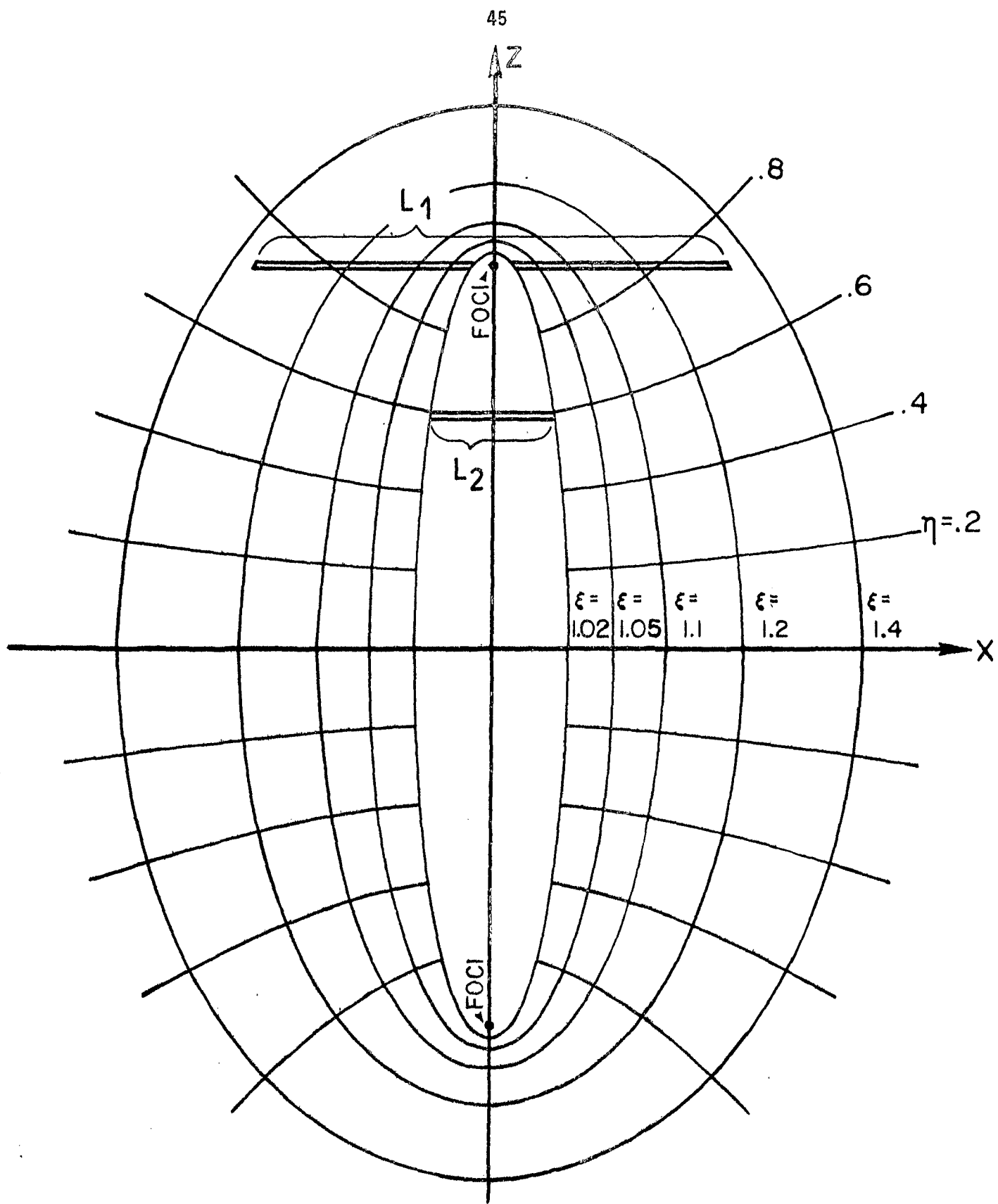


Figure 2.

$$\begin{aligned}
 \Phi_m(\xi, \eta, \phi) = & -\frac{ai}{2} F_Z P_1^1(\eta) P_1^1(\xi) \{ \cos \phi [(v-\bar{v})(1+C_1) + v(1+C_3)] \\
 & - \sin \phi [(u-\bar{u})(1+C_1) + u(1+C_3)] \} \\
 (10) \quad & + \frac{ai}{2} F_H W(1+C_2) P_1^1(\eta) P_1^1(\xi) \cos \phi \\
 & - \frac{a}{2} F_H U P_1^0(\eta) P_1^0(\xi) (1+C_4)
 \end{aligned}$$

where

$$\begin{aligned}
 c_1(\xi) &= - \frac{P_1^1(\xi_0) Q_1^1(\xi)}{Q_1^1(\xi_0) P_1^1(\xi)} \\
 c_2(\xi) &= \frac{Q_1^0(\xi_0) Q_1^1(\xi)}{i Q_1^1(\xi_0) Q_1^1(\xi_0) P_1^1(\xi)} \\
 c_3(\xi) &= - \left[\frac{P_1^1(\xi_0)}{Q_1^1(\xi_0)} - 2 \frac{Q_1^1(\xi_0) Q_1^1(\xi)}{P_1^1(\xi_0) P_1^1(\xi)} \right] \\
 c_4(\xi) &= - \frac{i P_1^1(\xi_0) Q_1^0(\xi)}{Q_1^1(\xi_0) P_1^0(\xi)}
 \end{aligned}$$

$$P_1^j = \frac{d}{d\xi} P_1^j$$

$$Q_1^j = \frac{d}{d\xi} Q_1^j$$

$$P_1^0(\xi) = \xi$$

$$i P_1^1(\xi) = (\xi^2 - 1)^{1/2}$$

$$Q_1^0(\xi) = \frac{1}{2\xi} \ln \frac{\xi+1}{\xi-1} - 1$$

$$Q_1^1(\xi) = (\xi^2 - 1)^{1/2} \frac{\xi}{\xi^2 - 1} - \frac{1}{2} \ln \frac{\xi + 1}{\xi - 1}$$

$$F_H = F_Y$$

u, v = velocity components of the water

U, V, W = velocity of instrument relative to the surrounding water.

Equation 10 defines the potential distribution about the body relative to the origin, $\xi = 1$ and $\eta = 0$. In reality one must measure the potential difference between two points. For our purposes we want to measure the potential difference between points on the same horizontal plane ($z = \text{constant}$) on opposite sides of the body. Let

$$(11) \quad \Delta\phi_m(\xi, \eta, \phi) \equiv \phi_m(\xi, \eta, \phi) - \phi_m(\xi, \eta, \phi + \pi)$$

also it is convenient to define $L = a i P_1^1(\xi) P_1^1(\eta)$
and divide Equation 10 by $F_z L (1 + C_1)$

then

$$(12) \quad \Delta\phi' \equiv \frac{\Delta\phi_m(\xi, \eta, \phi)}{F_z L (1 + C_1)} = -(\bar{v} - \bar{v} + C_5 V) \cos \phi + (u - u + C_5 V) \sin \phi + \frac{F_H}{F_z} C_6 W \cos \phi$$

where

$$C_5 \equiv (1+C_3)/(1+C_1)$$

$$C_6 \equiv (1+C_2)/(1+C_1)$$

Written in this way, Equation 12 shows more clearly the influence of the relative velocity V . As $C_5 \rightarrow 0$ the method measures u and v , the absolute velocity components, minus the vertically averaged flow. On the other hand as $C_5 \rightarrow 1$ the method measures $u + U$ and $v + V$, the velocity components of the instrument, minus \bar{u} and \bar{v} .

Numerical values of the C coefficients are given in Table 1 for $\xi_0 = (25/24)^{1/2}$ 1.02062 as function of ξ .

Suppose two potential difference measurements are made as shown in Figure 2. Let L_1 join the points $\xi_1 = 1.34, \eta_1 = 746$ and $\phi = \phi$ and $\phi + \pi$ and L_2 connect the points $\xi_2 = \xi_0, \eta_2 = 0.600$ and $\phi = \phi$ and $\phi + \pi$.

The point ξ_1, η_1 is out from the body a distance equal to 3 times the radius of the body at $\eta = 0$.

$$\begin{aligned} \text{Then } \Delta\Phi_1' &= -(v - \bar{v} + 0.947V) \cos \phi \\ &\quad + (u - \bar{u} + 0.947U) \sin \phi \\ &\quad + 0.970 \frac{F_H}{F_Z} W \cos \phi \end{aligned} \quad (13)$$

$$\begin{aligned} \Delta\Phi_2' &= -(v - \bar{v} + 0.106V) \cos \phi \\ &\quad + (u - \bar{u} + 0.106U) \sin \phi \\ &\quad + 0.500 F_H / F_Z W \end{aligned}$$

This example clearly demonstrates how different the measurements are when taken at different distances from the body. In case 1 the measurement is really of $\bar{v} - \bar{v} + V$ within an error of 5% of V while in the second case $\bar{v} - \bar{v}$ is measured to an error of 10% of V . However, these are not really errors for V can be determined from the difference between the measurements:

$$(14) \quad \Delta\Phi_1' - \Delta\Phi_2' = -0.841 (V \cos \phi - U \sin \phi) + 0.470 \frac{F_H}{F_Z} W \cos \phi$$

Table 1

Conversion coefficients C_1, C_2, C_3, C_5 and C_6 as functions of
for $\xi_0 = 25/24 = 1.0206\dots$

ξ	$Q'_1(\xi)/P'_1(\xi)$	C_1	C_2	C_3	C_5	C_6
1.0206...	22.203	0.894	-0.0529	-0.800	0.106	0.500
1.03	14.806	0.596	-0.0353	-0.533	0.293	0.605
1.05	8.387	0.338	-0.0120	-0.302	0.522	0.738
1.1	3.716	0.150	-0.0086	-0.134	0.753	0.862
1.2	1.528	0.062	-0.0036	-0.055	0.890	0.938
1.3	0.866	0.035	-0.0021	-0.031	0.936	0.964
1.34	0.720	0.029	-0.0017	-0.026	0.947	0.970
1.4	0.562	0.023	-0.0013	-0.020	0.958	0.976

It is important to examine the situation when the instrument becomes stationary on the sea floor. In this case $U = -u$, $V = -v$ and $W = 0$. Then

$$(15) \quad \Delta\Phi_1' = (\bar{v} - 0.053 v) \cos \phi - (\bar{u} - 0.053 u) \sin \phi$$

$$\Delta\Phi_2' = (\bar{v} - 0.894v) \cos \phi - (\bar{u} - 0.894u) \sin \phi$$

The $\Delta\Phi_1'$ measurement is influenced only at a level of 5% of the local velocity. So if the flow is less than 20 cm/s on the bottom the influence is less than 1 cm/s. However, as in the case of the falling probe the error term can be determined and, in principle, removed. The difference between the measurements is

$$(16) \quad \Delta\Phi_1' - \Delta\Phi_2' = 0.841 (v \cos \phi - u \sin \phi)$$

which is, of course, Equation 14 with $\tilde{V} = -v$.

The C coefficients have been computed with great precision but it should be emphasized that the theoretical analysis is an idealization of a actual instrument. The true coefficients will be different depending on the exact form or shape of the instrument, the various external attachments and on the validity of the theoretical analysis.

For $\phi = 0$ and $W = 0$ the response equations are

$$\Delta\Phi_1 = a_1 (v - \bar{v} + b_1 V); \quad \Delta\Phi_2 = a_2 (v - \bar{v} + b_2 V) .$$

A determination of a and b could be made by a series of measurements in a very large insulated tank. Suppose a small scale model of the instrument is constructed. Then when the model is placed in a tank in which a uniform electric current is applied the voltages sensed will be proportional to a. The measurement would best be performed using a low-frequency AC current in order to eliminate the DC bias of the electrodes.

A second experiment might be to tow the model at a known speed or oscillate it sinusoidally. From this experiment the product ab could be determined. Since a is known from the previous experiment then b can be calculated.

On the other hand, the theoretical coefficients may be known well enough to meet the system requirements. In this case, it would be appropriate to compare the EM profiler results with those from another system. However, the comparison should be performed against a more accurate method. Since the whole purpose here is to develop a superior method, the choice of a standard is critically important. The only better method available would be to acoustically track the probe as it falls over a high quality acoustic array. Strings of moored current meters and ship lowered profilers are not suitable for intercomparison work.

References

- Drever, R.G. and T.B. Sanford (1970) "A free-fall electromagnetic current meter-instrumentation", in: Proceeding of the IERE Conference on Electronic Engineering in Ocean Technology, Inst. of Elect. and Radio Engineers, London pp. 353-370.
- Drever, R.G. and T.B. Sanford (1976) "A velocity profiler based on acoustic Doppler principles", Technical Report, No. 76-96, Woods Hole Oceanographic Institution (unpublished manuscript).
- Düing, W. and D. Johnson (1972) "High resolution current profiling in the Straits of Florida", Deep-Sea Res., 19, pp. 259-274.
- Filloux, J.H., (1974) "Electric field recording on the sea floor with short span instruments" J.Geomag. & Geoelec. 26, 269-279.
- Müller, T.J., F.A. Schott, G. Siedler and K.P. Koltermann (1974) "Observations of overflow on the Iceland Faeroe Ridge. Meteor Forsch.-Ergebnisse, Reihe A, No. 15, 49-55.
- Osborn, T.R., (1974) "Vertical profiling of velocity microstructure" J. Phys. Oceanogr. 4 (1) pp. 109-115.
- Pochapsky, T.E., (1976) "Vertical structure of currents and deep temperatures in the Western Sargasso Sea" J. Phys. Oceanogr. 6 (1), pp. 45-56.
- Rossby, H.T., (1969) "A Vertical profile of currents near Plantagenet Bank", Deep-Sea Res. 16, pp. 377-385.
- Rossby, H.T., (1974) "Studies of the vertical structure of horizontal currents near Bermuda" J.Geophys. Res., 79, pp. 1781-1791.
- Sanford, T.B., (1971) "Motionally induced electric and magnetic fields in the sea" J. Geophys. Res., 76 (15), pp. 3476-3492.

Sanford, T.B., R.G. Drever, and J.H. Dunlap, (1974) "The design and performance of a free-fall electromagnetic velocity profiler (EMVP)", Ref. 74-46 Woods Hole Oceanographic Institution, Woods Hole, MA., (unpublished manuscript).

Siedler, G. and G. Krause (1964) " Ein System zur kontinuierlichen Messung physikalischer Größen in Meere" Kieler Meeresforschungen, 20 pp. 130-135.

Simpson, J.H., (1972) "A free fall probe for the measurement of velocity microstructure", Deep-Sea Res., 19, pp. 331-336.

Van Leer, J., W. Düing, R. Erath, E. Kennelly and A. Speidel (1974) "The Cycle-sonde: an unattended vertical profiler for scalar and vector quantities in the upper ocean", Deep-Sea Res., 21, pp. 385-400.