Appendix A – Tables with simulation results

A.1 Simulation results for single target setting

The statistical parameters for the single target setting were computed for each simulation scenario as follows:

$$\mathrm{M}_{bias} = \left(\tilde{\sigma}_{AG;EWC;mean} - \sigma_{bs;sim}\right) \times \frac{100\%}{\sigma_{bs;sim}} \qquad [1]$$

$$\tilde{\sigma}_{AG;EWC;mean} = \frac{1}{N_{sim}} \times \sum_{n=0}^{N_{sim}} \tilde{\sigma}_{AG;EWC;n} \qquad [2]$$

$$\mathrm{SD}_{M_{bias}} = \left(\frac{2 \times \mathrm{sd}}{\sqrt{N_{sim}}}\right) \times \frac{100\%}{\sigma_{bs;sim}} \qquad [3]$$

$$sd = \frac{1}{N_{sim}} \times \sum_{n=0}^{N_{sim}} \left(\tilde{\sigma}_{AG;EWC;n} - \tilde{\sigma}_{AG;EWC;mean}\right)^2 \qquad [4]$$

$$2\mathrm{SD} = 2 \times \mathrm{sd} \times \frac{100\%}{\sigma_{bs;sim}} \qquad [5]$$

$$\mathrm{P}_{i\%} = \left(p_{i\%} - \sigma_{bs;sim}\right) \times \frac{100\%}{\sigma_{bs;sim}} \qquad [6]$$

$$I_{\pm 2\mathrm{SD}} = \qquad [7]$$

Where,

M_{bias}	:	Measurement bias [% relative to $\sigma_{bs;sim}$]
$\tilde{\sigma}_{AG;EWC;mean}$	:	Mean measured $\tilde{\sigma}_{AG;EWC}$ [m²]
$\sigma_{bs;sim}$	:	True backscattering cross section of the simulated target (always one for the single target setting) [m²]

N_{sim} : Number of simulations runs
$2SD_{M_{bias}}$: Two times the standard error of $\tilde{\sigma}_{AG;EWC;mean}$ [% relative to $\sigma_{bs;sim}$]
sd : Standard deviation of the $\tilde{\sigma}_{AG;EWC}$ measurements [m²]
$2SD$: Two times the standard deviation of the $\tilde{\sigma}_{AG;EWC}$ measurements [% relative to $\sigma_{bs;sim}$]
$P_{i\%}$: Difference of the i-th percentile to $\sigma_{bs;sim}$] [% relative to $\sigma_{bs;sim}$]
$p_{i\%}$: i-th percentile of the $\tilde{\sigma}_{AG;EWC}$ measurements [m²]

Beam pattern setting	Gridding method	N_{sim}	M_{bias}	$2SD_{M_{bias}}$	2SD	$P_{0\%}$	$P_{25\%}$	$P_{50\%}$	$P_{75\%}$	$P_{100\%}$
Unshaded beam pattern	Block mean	811	+0.4 %	± 0.7 %	± 20.6 %	-17.4 %	-6.7 %	-4.2 %	+6.6 %	+32.3 %
	Weighted mean	811	-0.6 %	± 0.1 %	± 2.3 %	-3.4 %	-1.6 %	-0.5 %	+0.3 %	+1.8 %
Exp. shaded beam pattern	Block mean	825	+0.3 %	± 0.7 %	± 21.0 %	-17.9 %	-6.9 %	-4.2 %	+5.3 %	+42.7 %

	Weighted mean	825	-0.2 %	± 0.1 %	± 2.5 %	-2.6 %	-1.4 %	-0.2 %	+0.8 %	+2.5 %
Hann shaded beam pattern	Block mean	827	-0.4 %	± 0.6 %	± 17.3 %	-26.5 %	-6.5 %	-3.9 %	+4.9 %	+26.7 %
	Weighted mean	827	-0.0 %	± 0.1 %	± 2.4 %	-2.6 %	-1.1 %	+0.0 %	+1.1 %	+2.2 %

Table 1: Simulation results of different methods for the different beam pattern for the ideal vessel motion scenario using the limited volume $V_{Meas\,Opt}$ and approx. 800 simulation runs each

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Beam pattern setting	Voxel size	Vessel motion scenario	N_{sim}	M_{bias}	$2\mathrm{SD}_{M_{bias}}$	2SD	$\mathrm{P}_{0\%}$	$\mathrm{P}_{25\%}$	$\mathrm{P}_{50\%}$	$\mathrm{P}_{75\%}$	$\mathrm{P}_{100\%}$
Unshaded beam pattern	1 m	Ideal vessel motion	822	-0.8 %	± 0.2 %	± 4.9 %	-8.5 %	-2.6 %	-0.8 %	+1.1 %	+5.5 %
		Real vessel motion	831	-0.7 %	± 0.2 %	± 5.1 %	-9.9 %	-2.2 %	-0.7 %	+1.0 %	+14.7 %

Beam pattern setting	Voxel size	Vessel motion scenario	N_{sim}	M_{bias}	$2\mathrm{SD}_{M_{bias}}$	2SD	$\mathrm{P}_{0\%}$	$\mathrm{P}_{25\%}$	$\mathrm{P}_{50\%}$	$\mathrm{P}_{75\%}$	$\mathrm{P}_{100\%}$
		Exagg. vessel motion	862	-0.6 %	± 0.2 %	± 7.3 %	-24.6 %	-2.7 %	-1.0 %	+1.4 %	+20.3 %
	1.5 m	Ideal vessel motion	852	-0.7 %	± 0.1 %	± 2.7 %	-4.9 %	-1.6 %	-0.7 %	+0.5 %	+2.7 %
		Real vessel motion	829	-0.7 %	± 0.2 %	± 4.5 %	-8.6 %	-2.2 %	-0.7 %	+0.7 %	+7.1 %
		Exagg. vessel motion	801	-0.5 %	± 0.4 %	± 10.4 %	-22.7 %	-4.2 %	-1.1 %	+2.3 %	+21.4 %
	3 m	Ideal vessel motion	811	-0.6 %	± 0.1 %	± 2.3 %	-3.4 %	-1.6 %	-0.5 %	+0.3 %	+1.8 %
		Real vessel motion	825	-0.8 %	± 0.3 %	± 9.5 %	-26.8 %	-3.7 %	-0.9 %	+2.1 %	+21.2 %
		Exagg. vessel motion	827	-0.6 %	± 0.8 %	± 23.6 %	-29.7 %	-7.9 %	-2.3 %	+5.2 %	+56.1 %
Exp. shaded beam pattern	1 m	Ideal vessel motion	793	-0.2 %	± 0.2 %	± 4.9 %	-8.1 %	-2.0 %	-0.2 %	+1.6 %	+6.1 %
		Real vessel motion	834	-0.3 %	± 0.2 %	± 4.8 %	-7.5 %	-2.1 %	-0.3 %	+1.5 %	+6.6 %
		Exagg. vessel motion	786	-0.3 %	± 0.2 %	± 6.2 %	-12.7 %	-2.3 %	-0.3 %	+1.7 %	+13.8 %
	1.5 m	Ideal vessel motion	855	-0.1 %	± 0.1 %	± 2.6 %	-3.6 %	-1.2 %	-0.1 %	+1.1 %	+3.1 %

Beam pattern setting	Voxel size	Vessel motion scenario	N_{sim}	M_{bias}	$2\mathrm{SD}_{M_{bias}}$	2SD	$\mathrm{P}_{0\%}$	$\mathrm{P}_{25\%}$	$\mathrm{P}_{50\%}$	$\mathrm{P}_{75\%}$	$\mathrm{P}_{100\%}$
		Real vessel motion	784	-0.1 %	± 0.2 %	± 4.8 %	-10.2 %	-1.6 %	-0.1 %	+1.5 %	+7.3 %
		Exagg. vessel motion	795	+0.0 %	± 0.3 %	± 9.9 %	-12.8 %	-3.1 %	-0.6 %	+2.4 %	+22.0 %
	3 m	Ideal vessel motion	825	-0.2 %	± 0.1 %	± 2.5 %	-2.6 %	-1.4 %	-0.2 %	+0.8 %	+2.5 %
		Real vessel motion	827	-0.2 %	± 0.3 %	± 9.2 %	-16.2 %	-3.1 %	-0.4 %	+2.4 %	+15.2 %
		Exagg. vessel motion	800	+0.1 %	± 0.9 %	± 24.3 %	-42.1 %	-8.1 %	-1.1 %	+7.0 %	+74.8 %
Hann shaded beam pattern	1 m	Ideal vessel motion	819	-0.1 %	± 0.1 %	± 3.6 %	-5.9 %	-1.4 %	-0.2 %	+1.2 %	+4.9 %
		Real vessel motion	825	-0.1 %	± 0.1 %	± 3.9 %	-6.8 %	-1.5 %	-0.1 %	+1.2 %	+5.6 %
		Exagg. vessel motion	793	+0.1 %	± 0.2 %	± 4.6 %	-6.7 %	-1.5 %	+0.1 %	+1.6 %	+12.2 %
	1.5 m	Ideal vessel motion	808	-0.1 %	± 0.1 %	± 2.6 %	-3.2 %	-1.1 %	-0.2 %	+1.0 %	+2.5 %
		Real vessel motion	811	-0.0 %	± 0.1 %	± 3.8 %	-6.4 %	-1.2 %	-0.0 %	+1.3 %	+6.6 %
		Exagg. vessel motion	815	+0.0 %	± 0.3 %	± 7.3 %	-9.4 %	-2.4 %	-0.3 %	+2.3 %	+19.5 %

Beam pattern setting	Voxel size	Vessel motion scenario	N_{sim}	M_{bias}	$2SD_{M_{bias}}$	2SD	$P_{0\%}$	$P_{25\%}$	$P_{50\%}$	$P_{75\%}$	$P_{100\%}$
	3 m	Ideal vessel motion	827	-0.0 %	± 0.1 %	± 2.4 %	-2.6 %	-1.1 %	+0.0 %	+1.1 %	+2.2 %
		Real vessel motion	813	+0.1 %	± 0.3 %	± 7.6 %	-13.2 %	-2.4 %	-0.0 %	+2.5 %	+16.9 %
		Exagg. vessel motion	831	-0.1 %	± 0.6 %	± 18.4 %	-29.3 %	-6.9 %	-0.4 %	+6.2 %	+35.3 %

Table 2: Simulation results of different voxel sizes, for the different beam pattern and the different vessel motion scenarios using the limited volume $V_{Meas.Opt}$ and approx. 800 simulation runs each.

A.2 Simulation results for bubble stream setting

For the statistical analysis, all $\tilde{\sigma}_{AG;BL}$ values were normalized by the true aggregated backscattering cross section of all targets within the respective bubble stream layer (BL). The parameters were computed for each simulation scenario using the bubble stream setting as follows:

$$\tilde{\sigma}_{AG;BL;norm;n} = \frac{\tilde{\sigma}_{AG;BL;n}}{\sigma_{AG;BL;sim;n}} \quad [8]$$

$$\mathrm{M}_{bias;BL} = \left(\tilde{\sigma}_{AG;BL;mean} - 1\right) \times 100\% \quad [9]$$

$$\tilde{\sigma}_{AG;BL;mean} = \frac{1}{N_{sim;BL}} \times \sum_{n=0}^{N_{sim;BL}} \tilde{\sigma}_{AG;BL;norm;n} \quad [10]$$

$$\mathrm{SD}_{M_{bias;BL}} = \left(\frac{2 \times sd_{BL}}{\sqrt{N_{sim}}}\right) \times 100\% \quad [11]$$

$$sd_{BL} = \frac{1}{N_{sim;BL}} \times \sum_{n=0}^{N_{sim;BL}} \left(\tilde{\sigma}_{AG;BL;norm;n} - \tilde{\sigma}_{AG;BL;mean}\right)^2 \quad [12]$$

$$2\mathrm{SD}_{BL} = 2 \times \mathrm{sd}_{BL} \times 100\% \quad [13]$$

$$\mathrm{P}_{BL;i\%} = \left(p_{BL;i\%} - 1\right) \times 100\% \quad [14]$$

$$I_{\pm 2\mathrm{SD}} = \quad [15]$$

Where,

$\tilde{\sigma}_{AG;BL;norm;n}$	:	Normalized measured aggregated backscattering of layer BL of simulation run "n" $\left[\frac{m^2}{m^2}\right]$
$\tilde{\sigma}_{AG;BL;n}$	:	Measured aggregated backscattering of layer BL $\left(\tilde{\sigma}_{AG;BL}\right)$ of simulation run "n" [m²]
$\sigma_{AG;BL;sim;n}$	:	True aggregated backscattering cross section of the simulated targets within the water column layer BL during simulation run "n" [m²]
$\mathrm{M}_{bias;BL}$	:	Normalized measurement bias for the water column layer BL [% relative to 1]
$\tilde{\sigma}_{AG;BL;mean}$	:	Mean normalized $\tilde{\sigma}_{AG;BL}$ measurement for the bubble stream layer $\left[\frac{m^2}{m^2}\right]$
N_{sim}	:	Number of accepted simulation runs
$2\mathrm{SD}_{M_{bias;BL}}$	:	Two times the standard error of $\tilde{\sigma}_{AG;BL;mean}$ [% relative to 1]
sd_{BL}	:	Standard deviation of the normalized $\tilde{\sigma}_{AG;BL}$ measurements $\left[\frac{m^2}{m^2}\right]$
$2SD_{BL}$	:	Two times the standard deviation of the normalized $\tilde{\sigma}_{AG;BL}$ measurements [% relative to $\sigma_{AG;BL;sim}$]
$\mathrm{P}_{BL;i\%}$	:	Difference of the i-th percentile of the $\tilde{\sigma}_{AG;BL;norm}$ measurements to 1 [% compared to 1]
$\mathrm{p}_{BL;i\%}$	:	i-th percentile of the $\tilde{\sigma}_{AG;BL;norm}$ measurements $\left[\frac{m^2}{m^2}\right]$

Beam pattern setting	Voxel size	Vessel motion scenario	layer depth (BL)	N_{sim}	$M_{bias;BL}$	$2SD_{M_{bias;BL}}$	$2SD_{BL}$	$P_{BL;0\%}$	$P_{BL;25\%}$	$P_{BL;50\%}$	$P_{BL;75\%}$	$P_{BL;100\%}$
Unshaded beam pattern	1 m	Ideal vessel motion	42 m	580	-0.1 %	± 0.1 %	± 2.1 %	-3.0 %	-0.8 %	-0.1 %	+0.7 %	+2.2 %
			60 m	580	-0.3 %	± 0.1 %	± 1.9 %	-2.5 %	-1.0 %	-0.3 %	+0.4 %	+2.5 %
			108 m	580	-0.6 %	± 0.1 %	± 1.7 %	-2.8 %	-1.2 %	-0.6 %	+0.0 %	+2.0 %
		Real vessel motion	42 m	602	-0.1 %	± 0.1 %	± 2.2 %	-3.5 %	-0.9 %	-0.0 %	+0.8 %	+3.9 %
			60 m	602	-0.2 %	± 0.1 %	± 2.1 %	-3.0 %	-0.9 %	-0.1 %	+0.5 %	+2.4 %
			108 m	602	-0.5 %	± 0.1 %	± 2.0 %	-3.4 %	-1.1 %	-0.5 %	+0.2 %	+4.0 %
		Exagg. vessel motion	42 m	585	-0.2 %	± 0.1 %	± 2.8 %	-6.6 %	-1.1 %	-0.1 %	+0.7 %	+5.7 %
			60 m	585	-0.2 %	± 0.1 %	± 3.0 %	-4.1 %	-1.2 %	-0.2 %	+0.8 %	+6.2 %
			108 m	585	-0.5 %	± 0.2 %	± 3.7 %	-11.1 %	-1.7 %	-0.5 %	+0.7 %	+6.9 %
	1.5 m	Ideal vessel motion	42 m	562	-0.1 %	± 0.1 %	± 1.2 %	-1.7 %	-0.6 %	+0.0 %	+0.4 %	+1.2 %
			60 m	562	-0.2 %	± 0.0 %	± 1.1 %	-1.8 %	-0.6 %	-0.1 %	+0.2 %	+1.0 %

Beam pattern setting	Voxel size	Vessel motion scenario	layer depth (BL)	N_{sim}	$M_{bias;BL}$	$2SD_{M_{bias;BL}}$	$2SD_{BL}$	$P_{BL;0\%}$	$P_{BL;25\%}$	$P_{BL;50\%}$	$P_{BL;75\%}$	$P_{BL;100\%}$
			108 m	562	-0.6 %	± 0.1 %	± 1.2 %	-2.1 %	-1.0 %	-0.5 %	-0.1 %	+3.5 %
		Real vessel motion	42 m	565	-0.1 %	± 0.1 %	± 1.9 %	-3.5 %	-0.7 %	-0.0 %	+0.6 %	+3.7 %
			60 m	565	-0.1 %	± 0.1 %	± 2.1 %	-3.4 %	-0.9 %	-0.2 %	+0.6 %	+3.4 %
			108 m	565	-0.4 %	± 0.1 %	± 2.8 %	-4.4 %	-1.4 %	-0.4 %	+0.6 %	+4.7 %
		Exagg. vessel motion	42 m	597	-0.1 %	± 0.2 %	± 4.2 %	-5.9 %	-1.5 %	-0.2 %	+1.2 %	+6.8 %
			60 m	597	-0.3 %	± 0.2 %	± 5.1 %	-6.3 %	-2.0 %	-0.5 %	+1.3 %	+8.7 %
			108 m	597	-0.6 %	± 0.3 %	± 6.9 %	-8.7 %	-3.2 %	-1.0 %	+1.6 %	+13.5 %
	3 m	Ideal vessel motion	42 m	599	-0.1 %	± 0.1 %	± 1.6 %	-2.1 %	-0.7 %	-0.1 %	+0.5 %	+2.0 %
			60 m	599	-0.2 %	± 0.1 %	± 1.5 %	-2.1 %	-0.7 %	-0.1 %	+0.4 %	+1.6 %
			108 m	599	-0.3 %	± 0.1 %	± 2.0 %	-2.4 %	-0.9 %	-0.3 %	+0.3 %	+7.4 %
		Real vessel motion	42 m	630	+0.2 %	± 0.2 %	± 3.9 %	-6.5 %	-1.1 %	+0.1 %	+1.3 %	+7.0 %
			60 m	630	+0.1 %	± 0.2 %	± 4.8 %	-6.6 %	-1.5 %	+0.1 %	+1.6 %	+7.7 %

Beam pattern setting	Voxel size	Vessel motion scenario	layer depth (BL)	N_{sim}	$\mathrm{M}_{bias;BL}$	$2\mathrm{SD}_{M_{bias;BL}}$	$2SD_{BL}$	$\mathrm{P}_{BL;0\%}$	$\mathrm{P}_{BL;25\%}$	$\mathrm{P}_{BL;50\%}$	$\mathrm{P}_{BL;75\%}$	$\mathrm{P}_{BL;100\%}$
			108 m	630	+0.0 %	± 0.3 %	± 6.8 %	-9.9 %	-2.3 %	-0.3 %	+2.2 %	+11.9 %
		Exagg. vessel motion	42 m	613	+0.2 %	± 0.4 %	± 9.6 %	-13.1 %	-3.0 %	-0.0 %	+3.3 %	+15.6 %
			60 m	613	+0.1 %	± 0.5 %	± 12.5 %	-18.9 %	-4.3 %	-0.2 %	+4.3 %	+20.1 %
			108 m	613	+0.1 %	± 0.7 %	± 17.6 %	-23.9 %	-6.6 %	-0.1 %	+6.2 %	+26.6 %
Exp. shaded beam pattern	1 m	Ideal vessel motion	42 m	580	-0.1 %	± 0.1 %	± 2.1 %	-2.9 %	-0.9 %	-0.1 %	+0.7 %	+2.6 %
			60 m	580	-0.1 %	± 0.1 %	± 1.9 %	-2.4 %	-0.8 %	-0.1 %	+0.6 %	+2.4 %
			108 m	580	-0.2 %	± 0.1 %	± 1.5 %	-2.3 %	-0.8 %	-0.3 %	+0.2 %	+2.3 %
		Real vessel motion	42 m	612	-0.2 %	± 0.1 %	± 2.4 %	-4.5 %	-1.0 %	-0.1 %	+0.7 %	+2.6 %
			60 m	612	-0.2 %	± 0.1 %	± 2.1 %	-3.2 %	-0.9 %	-0.2 %	+0.6 %	+2.4 %
			108 m	612	-0.3 %	± 0.1 %	± 2.0 %	-3.4 %	-1.0 %	-0.3 %	+0.5 %	+2.9 %
		Exagg. vessel motion	42 m	572	-0.1 %	± 0.1 %	± 2.9 %	-4.8 %	-1.1 %	-0.1 %	+0.8 %	+5.3 %

Beam pattern setting	Voxel size	Vessel motion scenario	layer depth (BL)	N_{sim}	$\mathrm{M}_{bias;BL}$	$2\mathrm{SD}_{M_{bias;BL}}$	$2SD_{BL}$	$\mathrm{P}_{BL;0\%}$	$\mathrm{P}_{BL;25\%}$	$\mathrm{P}_{BL;50\%}$	$\mathrm{P}_{BL;75\%}$	$\mathrm{P}_{BL;100\%}$
			60 m	572	-0.2 %	± 0.1 %	± 3.3 %	-6.0 %	-1.3 %	-0.2 %	+0.9 %	+6.3 %
			108 m	572	-0.3 %	± 0.2 %	± 4.0 %	-8.6 %	-1.6 %	-0.2 %	+1.0 %	+6.1 %
	1.5 m	Ideal vessel motion	42 m	592	-0.1 %	± 0.1 %	± 1.2 %	-1.5 %	-0.5 %	+0.1 %	+0.4 %	+1.2 %
			60 m	592	-0.1 %	± 0.0 %	± 1.1 %	-1.6 %	-0.5 %	+0.1 %	+0.3 %	+1.1 %
			108 m	592	-0.1 %	± 0.1 %	± 1.3 %	-1.6 %	-0.6 %	-0.1 %	+0.3 %	+4.0 %
		Real vessel motion	42 m	594	-0.0 %	± 0.1 %	± 1.9 %	-2.7 %	-0.7 %	-0.0 %	+0.5 %	+3.0 %
			60 m	594	-0.1 %	± 0.1 %	± 2.2 %	-3.2 %	-0.8 %	-0.1 %	+0.7 %	+3.5 %
			108 m	594	-0.1 %	± 0.1 %	± 2.7 %	-4.3 %	-1.0 %	-0.2 %	+0.7 %	+4.9 %
		Exagg. vessel motion	42 m	601	+0.1 %	± 0.2 %	± 4.0 %	-6.0 %	-1.3 %	+0.1 %	+1.5 %	+7.2 %
			60 m	601	+0.1 %	± 0.2 %	± 5.1 %	-7.0 %	-1.7 %	+0.1 %	+1.9 %	+10.6 %
			108 m	601	+0.2 %	± 0.3 %	± 6.6 %	-9.2 %	-2.3 %	+0.1 %	+2.5 %	+9.2 %
	3 m	Ideal vessel motion	42 m	575	-0.1 %	± 0.1 %	± 1.5 %	-1.8 %	-0.6 %	-0.1 %	+0.5 %	+1.9 %

Beam pattern setting	Voxel size	Vessel motion scenario	layer depth (BL)	N_{sim}	$\mathrm{M}_{bias;BL}$	$2\mathrm{SD}_{M_{bias;BL}}$	$2SD_{BL}$	$\mathrm{P}_{BL;0\%}$	$\mathrm{P}_{BL;25\%}$	$\mathrm{P}_{BL;50\%}$	$\mathrm{P}_{BL;75\%}$	$\mathrm{P}_{BL;100\%}$
			60 m	575	-0.1 %	± 0.1 %	± 1.5 %	-2.1 %	-0.6 %	-0.1 %	+0.5 %	+2.1 %
			108 m	575	+0.1 %	± 0.1 %	± 2.2 %	-2.4 %	-0.5 %	-0.0 %	+0.5 %	+10.5 %
		Real vessel motion	42 m	592	-0.0 %	± 0.2 %	± 4.3 %	-6.8 %	-1.5 %	-0.0 %	+1.3 %	+7.6 %
			60 m	592	-0.0 %	± 0.2 %	± 5.1 %	-7.8 %	-1.6 %	-0.1 %	+1.7 %	+8.9 %
			108 m	592	-0.0 %	± 0.3 %	± 7.0 %	-12.0 %	-2.4 %	-0.1 %	+2.3 %	+11.5 %
		Exagg. vessel motion	42 m	592	-0.1 %	± 0.4 %	± 10.0 %	-18.5 %	-3.3 %	-0.1 %	+3.1 %	+16.2 %
			60 m	592	-0.2 %	± 0.5 %	± 12.9 %	-23.5 %	-4.7 %	-0.3 %	+4.5 %	+22.0 %
			108 m	592	-0.2 %	± 0.7 %	± 17.7 %	-29.4 %	-6.5 %	-0.0 %	+6.1 %	+30.5 %
Hann shaded beam pattern	1 m	Ideal vessel motion	42 m	601	-0.0 %	± 0.1 %	± 1.9 %	-2.3 %	-0.7 %	-0.0 %	+0.7 %	+2.5 %
			60 m	601	-0.0 %	± 0.1 %	± 1.7 %	-2.0 %	-0.6 %	+0.0 %	+0.6 %	+2.1 %
			108 m	601	+0.0 %	± 0.1 %	± 1.4 %	-1.9 %	-0.5 %	+0.0 %	+0.5 %	+2.9 %

Beam pattern setting	Voxel size	Vessel motion scenario	layer depth (BL)	N_{sim}	$M_{bias;BL}$	$2SD_{M_{bias;BL}}$	$2SD_{BL}$	$P_{BL;0\%}$	$P_{BL;25\%}$	$P_{BL;50\%}$	$P_{BL;75\%}$	$P_{BL;100\%}$
		Real vessel motion	42 m	581	+0.0 %	± 0.1 %	± 2.0 %	-2.9 %	-0.7 %	+0.1 %	+0.7 %	+2.7 %
			60 m	581	+0.0 %	± 0.1 %	± 1.9 %	-3.0 %	-0.6 %	+0.1 %	+0.7 %	+2.8 %
			108 m	581	+0.1 %	± 0.1 %	± 1.7 %	-2.4 %	-0.5 %	+0.0 %	+0.7 %	+3.4 %
		Exagg. vessel motion	42 m	586	-0.1 %	± 0.1 %	± 2.7 %	-4.1 %	-1.0 %	-0.1 %	+0.8 %	+5.2 %
			60 m	586	-0.1 %	± 0.1 %	± 2.7 %	-4.5 %	-0.9 %	-0.0 %	+0.8 %	+4.0 %
			108 m	586	-0.0 %	± 0.1 %	± 2.9 %	-6.9 %	-1.0 %	-0.0 %	+1.0 %	+6.7 %
	1.5 m	Ideal vessel motion	42 m	569	-0.1 %	± 0.0 %	± 1.2 %	-1.6 %	-0.5 %	+0.1 %	+0.4 %	+1.1 %
			60 m	569	-0.0 %	± 0.0 %	± 1.1 %	-1.5 %	-0.5 %	+0.1 %	+0.4 %	+1.0 %
			108 m	569	-0.0 %	± 0.0 %	± 1.1 %	-1.3 %	-0.4 %	+0.0 %	+0.4 %	+3.0 %
		Real vessel motion	42 m	612	-0.0 %	± 0.1 %	± 1.8 %	-3.6 %	-0.6 %	+0.0 %	+0.6 %	+2.5 %
			60 m	612	-0.0 %	± 0.1 %	± 1.9 %	-3.4 %	-0.6 %	+0.1 %	+0.7 %	+3.3 %
			108 m	612	+0.1 %	± 0.1 %	± 1.8 %	-3.0 %	-0.5 %	+0.0 %	+0.7 %	+3.1 %

Beam pattern setting	Voxel size	Vessel motion scenario	layer depth (BL)	N_{sim}	$M_{bias;BL}$	$2SD_{M_{bias;BL}}$	$2SD_{BL}$	$P_{BL;0\%}$	$P_{BL;25\%}$	$P_{BL;50\%}$	$P_{BL;75\%}$	$P_{BL;100\%}$
		Exagg. vessel motion	42 m	614	-0.0 %	± 0.2 %	± 3.8 %	-5.4 %	-1.3 %	-0.1 %	+1.2 %	+5.7 %
			60 m	614	-0.1 %	± 0.2 %	± 4.6 %	-7.3 %	-1.6 %	-0.1 %	+1.5 %	+6.5 %
			108 m	614	+0.0 %	± 0.2 %	± 4.6 %	-7.1 %	-1.7 %	+0.0 %	+1.7 %	+10.1 %
	3 m	Ideal vessel motion	42 m	561	-0.1 %	± 0.1 %	± 1.5 %	-2.0 %	-0.6 %	-0.0 %	+0.5 %	+1.8 %
			60 m	561	-0.0 %	± 0.1 %	± 1.5 %	-2.0 %	-0.5 %	-0.0 %	+0.5 %	+1.8 %
			108 m	561	+0.2 %	± 0.1 %	± 2.2 %	-2.0 %	-0.5 %	+0.1 %	+0.7 %	+10.6 %
		Real vessel motion	42 m	582	-0.1 %	± 0.2 %	± 3.9 %	-6.2 %	-1.4 %	-0.0 %	+1.3 %	+5.4 %
			60 m	582	-0.1 %	± 0.2 %	± 4.5 %	-7.4 %	-1.8 %	-0.1 %	+1.5 %	+6.6 %
			108 m	582	+0.1 %	± 0.2 %	± 5.1 %	-7.9 %	-1.7 %	+0.1 %	+1.8 %	+9.6 %
		Exagg. vessel motion	42 m	600	+0.1 %	± 0.4 %	± 9.9 %	-17.9 %	-3.5 %	+0.3 %	+3.3 %	+18.1 %
			60 m	600	+0.1 %	± 0.5 %	± 12.0 %	-21.7 %	-4.0 %	+0.2 %	+3.8 %	+19.0 %
			108 m	600	+0.3 %	± 0.5 %	± 12.5 %	-25.6 %	-4.0 %	+0.5 %	+4.3 %	+23.9 %

Table 3: Measurement results for the bubble stream setting. Only results for bubble streams within ±49 across-track distance from the ship track have been used to exclude boundary effects.

Threshold relative to $\boldsymbol{Max(\tilde{\sigma}_{Voxel})}$	**Absolute measurement bias for different beam pattern (%)**		
	Unshaded	*Exp.-shaded*	*Hann shaded*
-60 dB	**-0.7**	**-0.3**	**+0.0**
-50 dB	**-0.8**	**-0.4**	**+0.0**
-40 dB	-1.4	**-0.8**	**+0.0**
-30 dB	-3.8	-2.0	**-0.1**
-20 dB	-9.9	-4.8	**-0.9**
-13 dB	-17.8	-9.0	-5.0
-10 dB	-23.7	-14.5	-10.5
-6 dB	-40.6	-31.9	-27.7
-3 dB	-67.8	-61.2	-57.0

Table 4: $M_{bias;BL}$ *values computed from ~600 simulations for each beam pattern setting, after applying a threshold to the grids (dB compared to maximum* $\tilde{s}_{V;pn,sn,bn}$ *value within the layer). (108 m depth, 12m layer, 1 m voxel size).* $\left|M_{bias;BL}\right| \leq 1\%$ *is highlighted bold.*

Appendix B – Equivalent sample volume for mills cross MBESs

Coordinate system and mills cross beam pattern

The mills cross MBES transmit- and receive unit are modeled as two independent line arrays at the center of the ships coordinate system. The cartesian ship's coordinate system is described using x', y', z' to distinguish it from x, y, z coordinates of a world coordinate system that includes the ship. x' is defined positive forwards, the y' positive starboard and z' positive downwards. The angles around these axes $\left(\vartheta_{x'}, \vartheta_{y'}, \vartheta_{z'}\right)$ are defined according to the right-hand rule. $\vartheta_{x'}$ and $\vartheta_{y'}$ are 0 at the positive z',-axis. $\vartheta_{z'}$ is 0 at the positive x'-axis.

The transmit array of the MBES is aligned with the x'-axis of the ship, while the receive array is aligned with the y'-axis. The 2D beam pattern along the main axis of each line-array $\left(B_{tx0}^2\left(\vartheta_{y'}\right)\right.$ and $\left.B_{rx0;bn}^2(\vartheta_{x'})\right)$ is modelled using delay-and-sum beam forming where the elements are placed at $\frac{\lambda}{2}$ distance. It is assumed that the line array elements are perfectly isotropic in all angular directions. The angular responses of the arrays are then ambiguous with respect to rotations about the respective line array axes (e.g. Burdic, 1991). The sensitivity of the 3D beam pattern towards each position in 3D space can thus be described using only the along-track- (angle in the transmit beam pattern) and across-track- (angle in the receive beam pattern) angles which can be computed for each coordinate as follows:

$$\vartheta_{y'} = \arcsin\left(\frac{x'}{R}\right) \quad [16]$$

$$\vartheta_{x'} = \arcsin\left(\frac{-y'}{R}\right) \quad [17]$$

$$R = \sqrt{x'^2 + y'^2 + z'^2} \quad [18]$$

Where,

$\vartheta_{y'}$: Along-track angle / Angle within the 2D transmit beam pattern. [rad]
$\vartheta_{x'}$: Across-track angle / Angle within the 2D receive beam pattern. [rad]
R : Radial distance towards transducer [m^2]
x' : X coordinate in the vessel coordinate system (positive forward) [m]
y' : y coordinate in the vessel coordinate system (positive starboard) [m]
z' : z coordinate in the vessel coordinate system (positive downward) [m]

The monostatic two-way beam pattern of the combined transceiver unit is the product the orthogonal transmit- and receive beam pattern (equation [6]):

$$BF_{bn}(\vartheta_{y'}, \vartheta_{x'}) = B_{tx0}^2(\vartheta_{y'}) \times B_{rx0;bn}^2(\vartheta_{x'}) \quad [19]$$

Where,

$BF_{bn}(\vartheta_{y'}, \vartheta_{x'})$: Two-way beam response function in ϑ_y and ϑ_x space
$B_{tx0}^2(\vartheta_{y'})$: Transmit swath beam pattern (line array along the x-axis)
$B_{rx0;bn}^2(\vartheta_{x'})$: Receive beam pattern of beam bn (line array along the y-axis)

The angles $\vartheta_{y'}$ and $\vartheta_{x'}$ thus allow for an easy description of the sensitivity of each position in 3D space within the receive and transmit beam pattern. However, the integration over the 3D beam pattern is better understood for standard spherical coordinate angles θ and φ. To substitute $\vartheta_{y'}$ and $\vartheta_{x'}$ with θ and φ, we first convert equations [56]-[58] into cartesian coordinates:

$$x' = R \times \sin(\vartheta_{y'}) \quad [20]$$

$$y' = R \times \sin(-\vartheta_{x'}) \quad [21]$$

$$z'^2 = \sqrt{R^2 - x'^2 + y'^2} \quad [22]$$

The transformation between spherical and cartesian coordinates is defined by:

$$x' = R \times \sin(\theta) \times \cos(\varphi) \quad [23]$$

$$y' = R \times \sin(\theta) \times \sin(\varphi) \quad [24]$$

$$z' = R \times \cos(\theta) \quad [25]$$

Combining equations [60], [63] and [61], [65] reveals the transformations:

$$\vartheta_{y'} = \arcsin(\sin(\theta) \times \cos(\varphi)) \quad [26]$$

$$\vartheta_{x'} = arcsin(-\sin(\theta) \times \sin(\varphi)) \quad [27]$$

Which can be used to substitute $BF_{bn}(\vartheta_{y'}, \vartheta_{x'})$ for an equivalent $BF_{bn}(\theta, \varphi)$.

Equivalent sampling volume from beam- and pulse response function

The equivalent acoustic sampling volume is defined as integral over the hemisphere below the vessel, modulated with the 3D beam and pulse response functions (see section *A simple forward model for water column imaging MBESs*):

$$\mathrm{V}_{sn,bn} = \int_{R=0}^{\infty} \int_{\theta=0}^{2\pi} \int_{\varphi=0}^{\frac{\pi}{2}} R^2 sin(\varphi) \times RF_{sn}(R) BF_{bn}(\theta, \varphi) d\varphi d\theta dR \quad [28]$$

Where,

$\mathrm{V}_{sn,bn}$	:	Equivalent acoustic sampling volume of sample sn of beam bn
$RF_{sn}(R)$	:	Range response function of equation (equation [6])
$BF_{bn}(\theta, \varphi)$	:	Beam response function in θ, φ space (equation[8])
θ	:	Polar angle [rad]
φ	:	Azimuthal angle [rad]
R_{sn}	:	Range of sample sn [m]
c	:	Speed of sound underwater $\left[\frac{m}{s}\right]$

The integration over the range and angles can be separated into two factors:

$$\mathrm{V}_{sn,bn} = \Psi_{bn} \times \mathrm{X}_{sn} \quad [29]$$

$$\Psi_{bn} = \int_{\theta=0}^{\frac{\pi}{2}} \int_{\varphi=0}^{2\pi} sin(\varphi) \times BF_{bn}(\theta, \varphi) d\varphi d\theta \approx \Omega_{TX} \times \Omega_{RX;bn} \quad [30]$$

$$\mathrm{X}_{sn} = \int_{R=0}^{\infty} R^2 \times RF_{sn}(R) dR \approx {R_{sn}}^2 \times c \times \frac{T_{eff}}{2} \quad [31]$$

Where,

Ψ_{bn}	:	Equivalent beam angle of beam bn [rad]
X_{sn}	:	Range dependent part of the volume integral [m^3]

Using equations[66], [67], [69],[70] and [71], ψ_{bn}, X_{sn} and thus $V_{sn,bn}$ can be computed numerally for arbitrary $BF_{bn}(\vartheta_{y\prime}, \vartheta_{x\prime})$ and $RF_{sn}(R)$ functions.

Approximate solution

Assuming narrow a beam pattern that accumulates most of the beam's energy in a small section of the hemisphere, the equivalent beam angle is well approximated as the product of the receive- and transmit- 1D (main axis) equivalent beam angles:

$$(for\ \Omega_{TX}\ and\ \Omega_{RX;bn} \lll\ \pi) \qquad [32]$$

$$\psi_{bn} = \int_{\theta=0}^{\frac{\pi}{2}} \int_{\varphi=0}^{2\pi} sin(\varphi) \times BF_{bn}(\theta, \varphi) d\varphi d\theta \approx \Omega_{TX} \times \Omega_{RX;bn}$$

$$\Omega_{TX} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} B_{tx0}^{2}(\vartheta_{y\prime}) d\vartheta_{y\prime} \qquad [33]$$

$$\Omega_{RX;bn} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} B_{rx0;bn}^{2}(\vartheta_{x\prime}) d\vartheta_{x\prime} \qquad [34]$$

Where,

ψ_{bn}	:	Equivalent two-way beam angle of beam bn.
Ω_{TX}	:	Equivalent one-way 1D beam angle of the main axis transmit beam pattern [rad]
Ω_{RX}	:	Equivalent one-way 1D beam angle of the main axis receive beam pattern of beam bn [rad]

Similarly, assuming that the pulse function RF_{sn} covers only a small distance compared to the range of the sample, X_{sn} is well approximated by:

$$(for\ T_{eff} \times \frac{c}{2} \ll\ R_{sn}) \qquad [35]$$

$$X_{sn} = \int_{R=0}^{\infty} R^2 \times RF_{sn}(R) dR \approx R_{sn}{}^2 \times c \times \frac{T_{eff}}{2}$$

$$T_{eff} = \int_{0}^{\infty} p(t) dt \qquad [36]$$

This leads to compute $V_{sn,bn}$ using:

$$V_{sn,bn} \approx R_{sn}{}^2 \times \Omega_{TX} \times \Omega_{RX;bn} \times c \times \frac{T_{eff}}{2} \qquad [37]$$

The approximation error for these functions was investigated numerically. For the simulated pulse envelope (Hann-shaded) the approximation error of X_{sn} is < 0.13 % for samples at distance (R_{sn}) larger than $10 \times c \times \frac{T_{eff}}{2}$ and reduced to < 0.01% for samples at a distance larger than $36 \times c \times \frac{T_{eff}}{2}$. For the simulated scenarios $\left(c \times \frac{T_{eff}}{2} = 0.375m\right)$ these distances translate to 3.75m (<0.13% error) and 13.5 m (<0.01% error) from the transducer, respectively.

The approximation error of ψ_{bn} depends on the exact beam pattern and is increases at high beam steering angles. We investigated the beam pattern for the simulated shading function (unshaded, exp. shaded, and Hann shaded) at 60° degrees beam steering angle for different transducer sizes (different equivalent beam angles). For a 1°x1° flat transducer, as simulated in the assessment (128 transducer elements), the error is <0.1% for all simulated beam patterns (error at 60° degrees, unshaded ~ 0.09%, exp. shaded ~0.025%, Hann shaded ~-0.025%). For smaller transducers, the error increases depending on the beam pattern. For a transducer with only 16 elements (ca. 8° beam opening at the center 16° at 60° beam steering angle) the error would be <1.2% (unshaded), <0.07%, (exp. shaded) and < 0.65% (Hann shaded).

References

Burdic, W.S., 1991. Underwater acoustic system analysis, 2nd ed. ed, Prentice Hall signal processing series. Prentice Hall, Englewood Cliffs, N.J.