Adaptive coupling strategy for joint inversions that use petrophysical information as constraints

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Introduction

Joint inversion is a very powerful strategy to integrate different geophysical data sets. Model resolution is generally improved and final results are less ambiguous than the results from the individual methods. One critical issue, however, is to find an adequate strategy to link data sets from each method, if the methods are sensitive to different physical parameters. For subsurface conditions where all physical parameters strongly correlate with each other, one straight-forward strategy is to link the parameters in the inversion by implementing fixed relationships as derivatives in the Jacobian matrix (e.g. Vermeesch et al., 2009). This strong but rigid coupling usually provides high model resolution, but introduces mostly unpredictable errors if the real parameter relationships differ strongly from the parameter relationships in parts of the investigated region. Moreover, determination of an adequate relative weighting of the different data sets can be challenging.

We suggest a more flexible joint inversion scheme, in which inversion steps are performed separately and coupling of the individual inversions is achieved by additional constraints accounting for the parameter relationships. In this way relative weighting of the data sets is not required. To make convergence behavior more robust and to better handle deviations from the considered relationship, the strengths of the coupling of the three methods vary adaptively and independently from each other during the inversion process.

Joint inversion setup

For our joint inversion scheme the inversion steps of the three methods MT, seismic refraction tomography and gravity are performed separately. The inversions are coupled by using additional terms $\Phi^{MT}_{(c)}$ in the objective functions that account for the fixed parameter relationships:

$$\begin{aligned}
\Phi^{MT} &= \underbrace{\Phi^{MT}_{(d)}(\mathbf{m}^{res.})}_{\text{data terms}} + \underbrace{(\lambda^{MT})^2\Phi^{MT}_{(m)}(\mathbf{m}^{res.})}_{\text{smoothing terms}} + \underbrace{(\mu^{MT})^2\Phi^{MT}_{(c)}(\mathbf{m}^{res.},\tilde{\mathbf{m}}^{res.})}_{\text{coupling terms}} \longrightarrow min\\
\Phi^{seis.} &= \Phi^{seis.}_{(d)}(\mathbf{m}^{vel.}) + (\lambda^{seis.})^2\Phi^{seis.}_{(m)}(\mathbf{m}^{vel.}) + (\mu^{seis.})^2\Phi^{seis.}_{(c)}(\mathbf{m}^{vel.},\tilde{\mathbf{m}}^{vel.}) \longrightarrow min\\
\Phi^{grav.} &= \Phi^{grav.}_{(d)}(\mathbf{m}^{dens.}) + (\lambda^{grav.})^2\Phi^{grav.}_{(m)}(\mathbf{m}^{dens.}) + (\mu^{grav.})^2\Phi^{grav.}_{(c)}(\mathbf{m}^{dens.},\tilde{\mathbf{m}}^{dens.}) \longrightarrow min
\end{aligned}$$

To determine the coupling constraints that are associated with the terms $\Phi_{(c)}$, we calculate for all inversion cells projections $\tilde{\mathbf{m}}$ of the physical parameters $\mathbf{m}$ onto the considered relationship curve and afterwards the distances of the model parameters and their projections are minimized:

$$\Phi^{MT}_{(c)}(\mathbf{m}^{res.},\tilde{\mathbf{m}}^{res.}) = \sum_{j=1}^{N}(m_j^{res.}-\tilde{m}_j^{res.})^2, \quad \Phi^{seis.}_{(c)}(\mathbf{m}^{vel.},\tilde{\mathbf{m}}^{vel.}) = \sum_{j=1}^{N}(m_j^{vel.}-\tilde{m}_j^{vel.})^2 \quad \text{and} \quad \Phi^{grav.}_{(c)}(\mathbf{m}^{dens.},\tilde{\mathbf{m}}^{dens.}) = \sum_{j=1}^{N}(m_j^{dens.}-\tilde{m}_j^{dens.})^2$$

Because the projections $\tilde{\mathbf{m}}$ depend all on $\mathbf{m}^{res.}$, $\mathbf{m}^{vel.}$ and $\mathbf{m}^{dens.}$ at the same time the inversions are linked. The projection method we used is illustrated in Fig. 1.

Adaptive scheme to determine the coupling parameters

Adaption of the coupling parameters μ^{MT}, $\mu^{seis.}$ and $\mu^{grav.}$ is performed independent from each other during the inversion process. However, the way how adaption is implemented is exactly the same for all three methods. Therefore, as an example, we explain the procedure only for one of them.

The criteria we use to control the adaption of μ is very fundamental. It is based on the idea that the coupling constraint affects the convergence behavior of our objective function at each iteration k by the same amount. It states that the incremental change of the sum of the data and regularization terms of the objective function

$$\Delta\Phi^{Constr.,k}_{(d+m)} := (\Phi^{Constr.,k}_{(d)} + \lambda^2\Phi^{Constr.,k}_{(m)}) - (\Phi^{Constr.,k-1}_{(d)} + \lambda^2\Phi^{Constr.,k-1}_{(m)})$$

for our constrained inversion should correspond to a specified portion D (with $1.0 > D > 0.0$) of the same terms

$$\Delta\Phi^{Ref.,k}_{(d+m)} := (\Phi^{Ref.,k}_{(d)} + \lambda^2\Phi^{Ref.,k}_{(m)}) - (\Phi^{Constr.,k-1}_{(d)} + \lambda^2\Phi^{Constr.,k-1}_{(m)})$$

for a reference inversion without constraint ($\mu_k = 0$):

$$\Delta\Phi^{Constr.,k}_{(d+m)} = D\Delta\Phi^{Ref.,k}_{(d+m)} \qquad (1)$$

Because the criteria is not explicitly dependent on the coupling parameters μ an additional assumption is required to develop an adaptive scheme. Here, we assume that μ is approximately linear with the normalized incremental change of the objective function Ψ_l for a number of L successive iterations:

$$\mu_l \approx p^{(0)} + p^{(1)} \underbrace{\frac{\Delta\Phi^{Ref.,l}_{(d+m)} - \Delta\Phi^{Constr.,l}_{(d+m)}}{\Delta\Phi^{Ref.,l}_{(d+m)}}}_{=:\Psi_l} \quad \text{with} \quad l = k-L-1,...,k \qquad (2)$$

To update μ at the k-th iteration the discrepancy principle and the assumption are now combined as follow (see Fig. 2):

1. two inversions - one with and one without the coupling constraint - are performed.
2. forward calculations are conducted for both updated models and the associated terms of the objective functions $\Delta\Phi^{Constr.,k}_{(d+m)}$ and $\Delta\Phi^{Ref.,k}_{(d+m)}$ are determined.
3. a linear regression of coupling parameters $\mu_{\tilde{l}}$ and normalized incremental change of the objective functions $\Psi_{\tilde{l}}$ from a number of previous iterations $\tilde{l} = k - \tilde{L} - 1, ..., k$ is carried out.
4. the determined axis intercept $p_k^{(0)}$ and slope $p_k^{(1)}$ from the linear regression are used to calculate the coupling parameter μ_{k+1} for the next iteration by means of the formula

$$\mu_{k+1} = (1-D)p_k^{(1)} + p_k^{(0)} \quad ,$$

which is obtained by a combination of eq. (1) and eq. (2).

Steps 1) to 4) are repeated at each iteration.

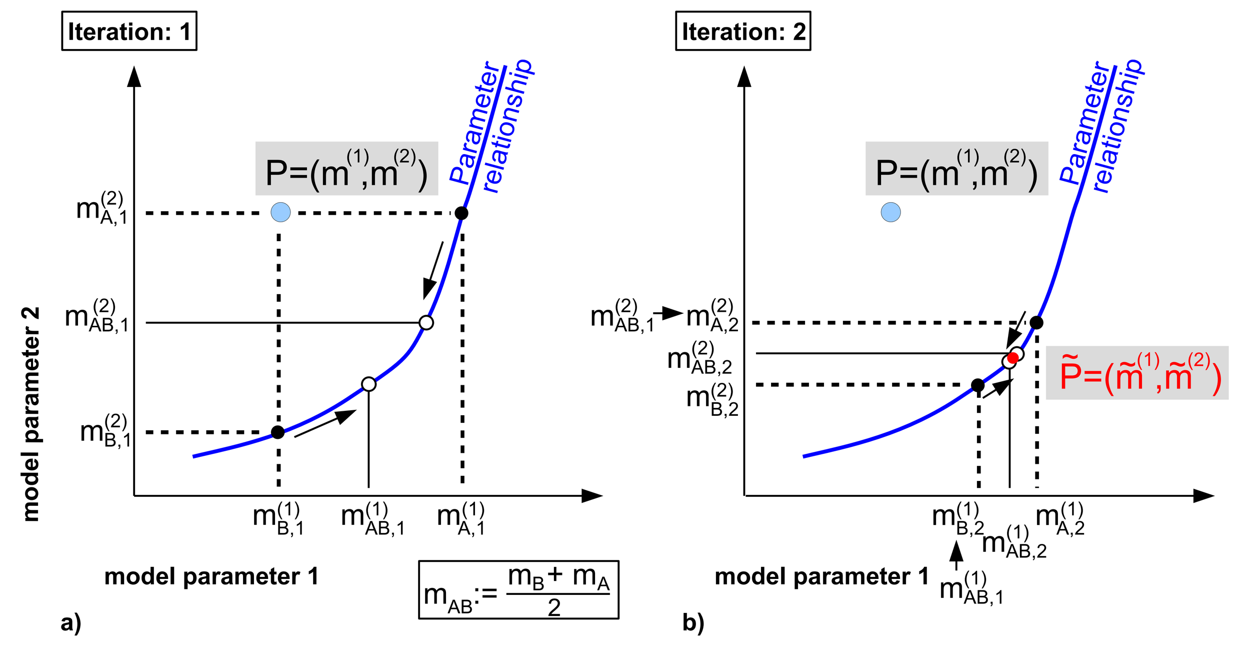

Fig.1: *Sketch illustrating the iterative procedure to determine for a point of two physical parameters $m^{(1)}$ and $m^{(2)}$ a projection $(\tilde{m}^{(1)}, \tilde{m}^{(2)})$ onto a relationship curve. (a) and (b) show the 1st and 2nd iteration step of the procedure.*

For the k-th iteration

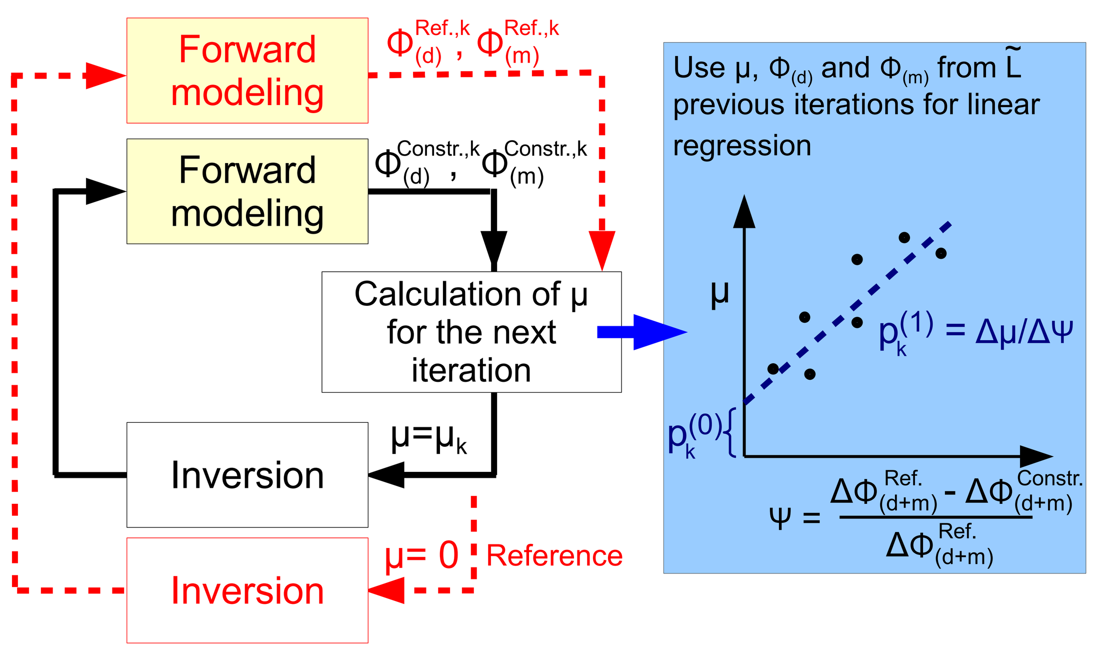

Fig.2: *Flowchart illustrating the adaptive inversion scheme for the k-th iteration. The axis intercept $p_k^{(0)}$ and slope $p_k^{(1)}$ determined from the regression of Ψ and μ from the previous $\tilde{L}$ iterations are used to update μ for the next iteration.*

Synthetic models

We test the adaptive scheme on a subbasalt model (see Fig.3; Column 1) and a salt model (Fig.7; Row 1). For both models rates of adaption of $D^{MT} = D^{seis.} = D^{grav.} = 0.1$ are chosen and information from 5 previous iterations are considered in the L_2-norm based linear regressions. At the first iteration coupling parameters are $\mu^{MT} = \mu^{seis.} = \mu^{grav.} = 0.25$. Regularization strengths are kept constant during the whole inversion process ($\lambda^{MT} = \lambda^{seis.} = \lambda^{grav.} = 0.5$).
First, we use parameter relationships that are valid everywhere in the models (see Fig.3 and Fig.7; Rows 1-3). Then the resistivity resp. density for one of the structures is lowered such that the relationships are not generally valid any more (see Fig.5 and Fig.7; Rows 4-5). For comparison, results from a joint inversion are shown for which parameter relationships are implemented as derivatives in the Jacobian matrix.

Model parameters:

Inversion:		
	Nr. of cells:	2312
	Cell sizes:	500 × 200 m
Data:		
MT:		
	Nr. of stations:	33
	Nr. of frequencies (both TE and TM mode):	15
	Frequency range:	$2.5 \cdot 10^{-5}$- 1 Hz
Seismic:		
	Nr. of shots/receivers:	177/34
	Nr. of rays:	6018
Gravity:		
	Nr. of stations:	60

Added data errors:

MT:	2% of abs. data values
Seismic:	10 ms
Gravity:	0.05 mgal

Starting models:

The final model derived from an individual seismic inversion (followed by an individual gravity inversion) is used as starting model for all inversions. However, various tests show that results of the adaptive joint inversion scheme are largely independent of the starting models.

Resistivities, velocities and densities in the synthetic models are related to each other by fixed parameter relationships.