Ocean-based Negative Emission Technologies

OCEAN NETs

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Author(s)	**Felix Meier**, **Wilfried Rickels**, **Christian Traeger**, Martin Quaas
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Title:
Carbon dioxide removal in a global analytic climate economy

Abstract:
Net-zero climate policies foresee deployment of atmospheric carbon dioxide removal with geological, terrestrial, or marine carbon storage. To study the effects of carbon dioxide removal (CDR) on the global economy and climate change mitigation efforts, we compare the global climate economy with CDR technologies available to a global climate economy without CDR. We find that with CDR net energy input and net emissions are lower over then entire time path. CDR affects the Social Cost of Carbon (SCC) via changes in total economic output, but has no direct effect on the analytic structure of the SCC. With CDR, the SCC is lower at the beginning, and higher in later years; carbon dioxide emissions are first higher and then lower. We show that the general description of the carbon cycle with N boxes allows for investigation of regional CDR deployment scenarios, reflecting different regional background conditions, different cost functions, and also different attitudes towards ocean CDR, and of the implications of variation in the carbon storage depth. The paper provides the basis for the analysis of decentralized and potentially non-cooperative CDR policies.

1. Introduction

1.1 Context

OceanNETs is a European Union project funded by the Commission's Horizon 2020 program under the topic of Negative emissions and land-use based mitigation assessment (LC-CLA-02-2019), coordinated by GEOMAR | Helmholtz Center for Ocean Research Kiel (GEOMAR), Germany.

OceanNETs responds to the societal need to rapidly provide a scientifically rigorous and comprehensive assessment of negative emission technologies (NETs). The project focuses on analyzing and quantifying the environmental, social, and political feasibility and impacts of ocean-based NETs. OceanNETs will close fundamental knowledge gaps on specific ocean-based NETs and provide more in-depth investigations of NETs that have already been suggested to have a high CDR potential, levels of sustainability, or potential co-benefits. It will identify to what extent, and how, ocean-based NETs can play a role in keeping climate change within the limits set by the Paris Agreement.

1.2 Purpose and scope of the deliverable

WP1 will contribute to understanding and assessing the (future) role of ocean-based negative emission technologies in climate policies. The aim of the task is to provide information on the possible future contribution of ocean NETs in different climate policies, i.e. globally coordinated and non-coordinated climate policies. D1.5 is supposed to facilitate the analysis of strategic interaction of ocean NETs, providing the basis for D1.7 which is suppose the synthesize the insights regarding the role of ocean NETs in climate policy. Hence, for D1.5 we developed a linear in-state IAM. Part of this model is a linear carbon cycle model (as in the original DICE specification) which is not restricted in the number of carbon cycle boxes. Hence the model allows for investigation various specific ocean NETs deployment questions, like for example local oceanNETs at the shore of the deploying country (affecting the upper ocean in other parts via mixing and feedback) or variation in injection depth. The original DICE specification would only allow to differentiate between upper and deep ocean carbon storage (achieved via ocean NETs) while the augmented model show how increasing depth of carbon storage (in the lower box) is beneficial and how much more one would be willing to pay for increasing storage depth. In general, the model provides a general framework for the analysis of ocean NETs which will be augmented for the case in question. We demonstrate this by showing the quantitative results for the original DICE specification (three boxes) and then demonstrate the insights obtained by horizontal differentiation (i.e. splitting the upper ocean box in various boxes to investigate regional deployment and strategic interaction) and vertical differentiation (i.e. splitting the lower box to investigate the implications of storage depth).

1.3 Relation to other deliverables

The developed model is supposed to provide insights for D1.6 which is supposed to look a region-specific damage (which could be ecosystem specific) resulting from ocean NETs deployment (or limiting their deployment). Together with D1.6, building on D1.2, D1.5 is supposed to provide the basis for the analysis of oceanNETs in climate policies in D1.7.

2. Technical part of the deliverable (choose your own headers)

The paper integrates ocean NETs (referred to as ocean CDR in the paper) into an analytical integrated assessment model with the aim to have a rich presentation of boxes to provide a stylized representation of the various marine CDR options. While exiting carbon cycle box models in existing integrated assessment models are restricted to a vertical differentiation of ocean carbon (i.e. atmosphere, upper ocean, deep ocean), this model also allows for a horizontal differentiation to investigate both coastal and regional differentiated ocean NETs deployment (i.e. country A engages in marine storage in its coastal carbon box, affecting the ocean social cost of carbon of other boxes and hence states as well) and for a richer representation of the vertical differentiation (i.e. investigation how carbon storage depth increases the benefits of ocean NETs deployment).

3. Conclusion

The focus of D1.5 has been adjusted in particular to reflect different ocean-NETs in a linear-in state model. D1.5 provides jointly with D1.6 the technical framework for analyzing ocean NETs in integrated assessment models which will applied in D1.7 to derive a synthesis of oceanNETs deployment in climate policies from an integrated assessment modelling perspective.

Carbon dioxide removal in a global analytic climate economy

Felix Meier[a], Wilfried Rickels[a], Martin Quaas[b], and Christian Traeger[c]

[a]Kiel Institute for the World Economy, Germany

[b]German Center for integrative Biodiversity Research (iDiv) Halle-Jena-Leipzig, Germany

[c]Department of Economics, University of Oslo, Norway

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Abstract: Net-zero climate policies foresee deployment of atmospheric carbon dioxide removal with geological, terrestrial, or marine carbon storage. To study the effects of carbon dioxide removal (CDR) on the global economy and climate change mitigation efforts, we compare the global climate economy with CDR technologies available to a global climate economy without CDR. We find that with CDR net energy input and net emissions are lower over then entire time path. CDR affects the Social Cost of Carbon (SCC) via changes in total economic output, but has no direct effect on the analytic structure of the SCC. With CDR, the SCC is lower at the beginning, and higher in later years; carbon dioxide emissions are first higher and then lower. We show that the general description of the carbon cycle with N boxes allows for investigation of regional CDR deployment scenarios, reflecting different regional background conditions, different cost functions, and also different attitudes towards ocean CDR, and of the implications of variation in the carbon storage depth. The model provides the basis for the analysis of decentralized and potentially non-cooperative CDR policies.

Keywords: carbon dioxide removal, climate change, integrated assessment, social cost of carbon, optimal carbon tax

1 Introduction

In line with the Paris Agreement to limit global warming to well below 2°C over preindustrial many countries have declared their intention to transition towards a net-zero emissions economy by the second half of this century (Tanaka and O'Neill, 2018). To accomplish this goal, technologies that remove carbon dioxide from the atmosphere with subsequent terrestrial, geological or marine carbon storage have been proposed (carbon dioxide removal, CDR). Furthermore, also capturing carbon at emissions point sources like industrial installations is discussed, requiring similar carbon storage options (Anderson and Newell, 2004). Almost all scenarios of future (net) greenhouse gas emissions that are consistent with the Paris Agreement include CDR options (IPCC, 2022). However, there are major concerns with the use of CDR technologies. A first concern is that the availability of CDR as an 'end of pipe' technology to clean up after greenhouse gas emissions, may be perceived as a substitute for conventional emission mitigation, which might lead to rebound effects (e.g. Geden et al., 2019), such that overall the effect on climate change might be limited. A second rebound effect may come about, as the process of capturing, transporting and storing carbon consumes additional energy and thus potentially leads to new emissions (IPCC, 2005). Third, in few storage sites, CO_2 will be permanently locked away. For most terrestrial and marine storage sites, there will be some kind of leakage. For this reason, the value of capturing and storing CO_2 may be less than the value of mitigating the emissions. This paper proposes an analytical framework for an integrated assessment of all these arguments around the use of CDR technologies.

The analytic integrated assessment model we propose includes the trade-off between mitigation of carbon emissions and CDR as well as the opportunity costs of energy required to capture and sequester carbon. We derive the optimal level of CDR deployment and analyze how emissions, energy input, and the social cost of carbon (SCC, equivalent to the carbon tax in a first-best economy) are affected by the introduction of CDR. Third, we derive equations for the shadowprices of CO_2 stored in alternative reservoirs, which quantify the costs of eventual leakage of CO_2 back to the atmosphere, and relate it to the social cost of carbon (SCC, equivalent to the carbon tax in a first-best economy).

Atmospheric carbon dioxide only represents a small fraction of the total carbon stock in the Earth System, namely 829 gigatons (Gt) out of a total of more than 45,696 gigatons (IPCC, 2013). The rest of carbon on earth is bound in other reservoirs. Especially the ocean has served as a major carbon sink over the past 200 years (Sabine et al., 2004). Due to the large storage capacity, the ocean has been suggested to be used as carbon storage achieved either by direct, intentional injection of carbon dioxide via ships or pipelines (Rickels and Lontzek, 2012), or by indirectly increased marine carbon uptake achieved by coastal blue carbon approaches, increasing marine biological productivity by restoring ecosystems (Bertram et al., 2021), via fertilization achieved for example by artificial up-welling, or by increasing the chemical buffer capacity of the ocean by adding alkaline materials (ocean alkalinity enhancement).

Whether a geological reservoir, such as an exploited oil field, is well suited for CDR is mainly determined by the rate at which carbon leaks back to the atmosphere (van der Zwaan and Gerlagh, 2009). A similar problem arises, if carbon is stored in the ocean. Due to feedback and saturation effects in the carbon cycle, some of the carbon that is injected into the oceans will eventually return to the atmosphere. CDR technologies are routinely included in numerical integrated assessment models. Rickels et al. (2018) study how well these effects are captured in currently used Integrated Assessment Models (IAMs). Rickels and Lontzek (2012) explore the economic implications of the ocean's imperfect storage property. They show that optimally each ton of carbon sequestered to the ocean is taxed at a rate lower than the optimal carbon tax for atmospheric carbon emission. In this paper, We derive the SCC, which quantifies the optimal tax on carbon emissions, for different reservoir types and analyze how the optimal carbon tax is affected by the introduction of CDR technologies by comparing the results of model specifications with and without the availability of CDR.

The paper is based on the recently emerging literature on analytic IAMs which have the feature that the SCC can be written as a constant fraction of total economic output (e.g. Traeger, 2022; Gerlagh and Lsiki, 2018; Golosov et al., 2014). This result arises from specifications of utility and climate damages which ensure that the climate-economy model is linear in the model's state variables, in particular human-made capital and the stocks of carbon in the different reservoirs (Karp, 2017; Traeger, 2022). We show that due to the linear-in-states property of analytic IAMs the deployment of CDR technologies has no effect on the analytic structure of the SCC. However, CDR alters the time path of total economic output and therefore influences the level of the SCC.

The paper is structured as follows. The next section introduces the option of CDR in an analytic climate-economy model. Section 3 presents the theoretical results on optimal emissions, CDR deployment, and the SCC, and compares them to the outcome of a standard climate-economy model without the option of CDR. The last section provides a numerical simulation for calibrated versions of both model types, showing then the implications of a more detailed carbon cycle for the analysis of ocean CDR.

2 Analytic climate-economy model

This section introduces CDR and the storage of carbon in different reservoirs into an analytic integrated assessment model of climate change. The underlying integrated assessment model is based on Golosov et al. (2014) and Traeger (2022). We consider a global economy where gross output Y_t is a function of technology A_t, capital K_t, labor N_t, and net energy input I_t,

$$Y_t = A_t \, K_t^{\kappa} \, N_t^{1-\kappa-v} \, I_t^{v} \qquad \text{with } K_0 > 0 \text{ given.} \tag{1}$$

The subscript zero denotes that technology and labor are prescribed by time dependent exogenous processes, as in the DICE model.

Subscript t denotes time. We distinguish between net energy I_t entering production and gross energy E_t, which also includes the energy used for CDR. The overall energy production E_t uses an exhaustible resource stock R_t, which is an aggregate of fossil fuels (coal, oil, and natural gas). We follow Golosov et al. (2014) and measure energy input in terms of its carbon content (in GtC). Absent carbon capture, E_t is directly equal to carbon entering the atmosphere. We also measure the resource stock R_t by its carbon content implying the equation of motion

$$R_{t+1} = R_t - E_t, \qquad \text{with } R_0 > 0 \text{ given.} \tag{2}$$

Following Traeger (2022), we allow for a finite number of carbon reservoirs ("boxes") with carbon contents, $M_{1,t}, \ldots, M_{r,t}$ with $r \in \mathbb{N}$. The first reservoir $M_{1,t}$ represents the atmospheric stock of carbon. The other boxes reflect the carbon stocks of the different layers of the ocean and the biosphere, and potential geological storage capacities. There are no direct capacity constraints for any of these reservoirs; for a discussion of such constraints see, e.g., Lafforgue et al. (2008). A higher amount of carbon in any of the reservoirs increases spillovers to other reservoirs. The carbon dynamics follows a standard linear carbon cycle model

$$\begin{pmatrix} M_{1,t+1} \\ M_{2,t+1} \\ \vdots \\ M_{r,t+1} \end{pmatrix} = \begin{pmatrix} \phi_{11} & \ldots & \phi_{r1} \\ \phi_{12} & \ldots & \phi_{r2} \\ \vdots & \ddots & \vdots \\ \phi_{1r} & \ldots & \phi_{rr} \end{pmatrix} \begin{pmatrix} M_{1,t} \\ M_{2,t} \\ \vdots \\ M_{r,t} \end{pmatrix} + \begin{pmatrix} E_t^{net} + E_t^{\text{exo}} \\ G_{2,t} \\ \vdots \\ G_{r,t} \end{pmatrix}, \tag{3}$$

or, in matrix notation

$$\boldsymbol{M}_{t+1} = \boldsymbol{\Phi} \boldsymbol{M}_t + \boldsymbol{E_t}.$$

where the transition matrix $\boldsymbol{\Phi}$ characterizes the carbon flows between the reservoirs.

CDR technologies remove carbon from the atmosphere decreasing $M_{1,t}$ and storing it in another reservoir $M_{i,t}$ with $i \in \{2, ..., r\}$. We let $G_{i,t}$ denote the amount of carbon (GtC) that CDR technologies remove from the atmosphere and push into reservoir i during period t, where $i \in \{2, ..., r\}$. As long as (gross) emissions are positive our technologies removing carbon from the atmosphere can also include carbon capture and storage. Net emissions are the difference between the carbon emitted during the production process and the carbon removed into other reservoirs

$$E_t^{net} = E_t - \sum_{i=2}^{r} G_{i,t} \tag{4}$$

The total amount of carbon entering or leaving the atmosphere is the sum of net emissions E_t^{net} and emissions from exogenous processes including land use change and forestry, which we denote by E_t^{exo}.

CDR's major operational cost is its energy consumption. Its energy consumption is also CDR's major downside when it comes to reducing CO_2 emissions. To flesh out this trade-off, we measure all operational costs $f_i(G_{i,t})$ in energy equivalents, which allows us to derive a nice and intuitive formula for the CDR deployment.[1] We define the net energy input into production as fossil-based energy less the energy used for CDR

$$I_t = E_t - \sum_{i=2}^{r} f_i(G_{i,t}). \tag{5}$$

If a storage reservoir is not used, the corresponding costs of CDR are zero,

[1]The dynamic programming model still solves analytically for a set of CDR sectors explicitly using labor and capital. However, each period's labor distribution across sectors generally has to be solved numerically, defying a similarly insightful derivation of the results.

$f_i(0) = 0$. We assume that marginal costs are positive and increasing for all storage units, $f_i'(G_{i,t}) > 0$ and $f_i''(G_{i,t}) > 0$. A reduction of atmospheric carbon emissions can either be achieved by reducing the energy input E_t directly (mitigation) or by using CDR ($G_{i,t}$). Since the cost for mitigation and CDR deployment can both be measured in energy units (in GtC), reservoir i will only be used if its cost (and marginal cost) is lower than the cost (and marginal cost) of mitigation, thus $f_i(G_{i,t}) \leq G_{i,t}$, and $f_i'(G_{i,t}) \leq 1$. As a result of measuring energy in CO_2 Note that without the option of CDR, net energy input, emissions, and net emissions are equivalent, $I_t = E_t = E_t^{net}$.

We follow Golosov et al. (2014) and assume a direct mapping of climate change damages from the atmospheric carbon stock $M_{1,t}$. The damage function, that shows climate damage as a fraction of gross output, is given by

$$D_t(M_{1,t}) = 1 - \exp\left[-\xi_0\left(M_{1,t} - M_1^{pre}\right)\right], \tag{6}$$

where M_1^{pre} denotes the pre-industrial atmospheric carbon concentration. The climate change damage parameter $\xi_0 > 0$ scales the marginal climate damage of atmospheric carbon, and can be reasonably calibrated to the climate damages in the DICE model (see e.g. Golosov et al., 2014).

Output net climate change damages is therefore given by $Y_t^{net} = Y_t\,\left[1 - D_t\left(M_{1,t}\right)\right]$. The model does not include any impacts from increasing carbon concentrations in the ocean (e.g. from ocean acidification).

Following Golosov et al. (2014) we assume full depreciation of capital over the course of 10 years, and the model's time step is chosen accordingly to equal 10 years. Thus, the economy's capital stock in the next period is given as the difference between net output Y_t^{net}, and consumption C_t,

$$K_{t+1} = Y_t \left[1 - D_t\left(M_{1,t}\right)\right] - C_t \tag{7}$$
$$= Y_t \exp\left[-\xi_0\left(M_{1,t} - M_1^{pre}\right)\right] - C_t.$$

The consumption rate is defined as $x_t = \frac{C_t}{Y_t^{net}}$, such that $1 - x_t$ is the savings rate.

We solve the model for a social planner who maximizes the present value of welfare from an infinite stream of consumption flows,

$$\max_{x_t, E_t, G_{i,t}} \sum_{t=0}^{\infty} \beta^t \log(C_t), \tag{8}$$

by choosing the consumption rate, emissions, and CDR deployment, subject to the constraints imposed by the economy and the climate system, equations (1) to (7). In (8), the parameter β denotes the utility discount factor.

3 Theoretical results

This section presents the results of the climate-economy model, and compares them to the outcome of an alternative model specification without a CDR technology.

3.1 Carbon dioxide removal

Appendix A solves the intertemporal optimization problem. It shows that the optimal rate of consumption is constant over time, $x_t^* = 1 - \beta\,\kappa$, and that the shadow value of the fossil resource stock, denoted by $\varphi_{R,t}$, monotonically grows over time according to Hotelling's (1931) rule, $\varphi_{R,t} = \beta^{-t}\varphi_{R,0}$. In the following, we summarize the results on optimal CDR deployment.

Proposition 1. *The optimal level of CDR deployment for reservoir i is given by*

$$G^*_{i,t} = f_i'^{-1}\left(\frac{\beta\,\xi_0\left[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}\right]_{1,1} - \beta\,\xi_0\left[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}\right]_{1,i}}{\beta\,\xi_0\,\left[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}\right]_{1,1} + (1-\beta\,\kappa)\beta^{-t}\varphi_{R,0}}\right), \tag{9}$$

where $[\cdot]_{1,1}$ *denotes the first, and* $[\cdot]_{1,i}$ *denotes the* i^{th} *element of the first column of the inverted matrix in square brackets. Note that the inverse of the marginal cost function is expressed by* $f_i'^{-1}$ *and that* $\left[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}\right]_{1,1} > \left[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}\right]_{1,i}$.

Proof. See Appendix B. □

Optimal CDR deployment is a function of constant model parameters, and the endogenously determined shadow value of the resource stock, which monotonically grows over time. Since $f_i'(G_{i,t})$ is an increasing function, also its inverse $f_i'^{-1}$ is an increasing function. Thus, optimal CDR immediately starts with its maximum level and then monotonically declines over time.

The interpretation of the carbon dynamics contributions follows Traeger (2022): The term $\left[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}\right]_{1,1}$ characterizes the discounted sum of carbon persisting in and returning to the atmospheric carbon stock in all future periods. The term $\left[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}\right]_{1,i}$ characterizes the long-term contribution to the atmospheric carbon reservoir from carbon that is currently stored in reservoir i. This 'leakage' determines the optimal amount of carbon stored in reservoir i. Everything else equal, it is optimal to store less carbon in a reservoir with a higher amount of leakage.

The numerator in equation (9) shows the marginal benefit of the new technology. CDR reduces the marginal damage of emissions as it allows to remove carbon from the atmosphere and store it in a less damaging reservoir i. The denominator shows the marginal cost of fossil energy. It captures the opportunity cost of the resource and the marginal damage that it creates.

The magnitude of the benefit from CDR is determined by the difference in the carbon dynamics contributions of the atmosphere and reservoir i. A decrease in the carbon persistence of reservoir i increases its carbon dynamics contribution as more carbon eventually finds its way into the atmosphere. This decreases the marginal benefit of CDR, and hence G_t^* declines. In contrast, an increase in the climate change damage parameter ξ_0 or an increase in the atmospheric carbon dynamics contribution $[(\mathbf{1} - \beta\, \mathbf{\Phi})^{-1}]_{1,1}$ raises the marginal damage of emissions and makes CDR technologies more attractive.

3.2 Emissions and energy input

Using the solution for CDR deployment allows to derive the optimal levels for emissions, and net energy input.

Proposition 2. *Optimal carbon emissions into the atmosphere are given by*

$$E_t^* = \frac{v}{\beta\, \xi_0\, [(\mathbf{1} - \beta\, \mathbf{\Phi})^{-1}]_{1,1} + (1 - \beta\, \kappa)\beta^{-t}\varphi_{R,0}} + \sum_{i=2}^{r=4} f_i \left(G_{i,t}^*\right), \tag{10}$$

with optimal CDR deployment $G_{i,t}^*$ as defined in equation (9).

Proof. See Appendix C. □

Optimal emissions are given by the sum of two terms. The first term captures the marginal benefit (numerator) and the marginal cost (denominator) from fossil energy. The term monotonically declines over time as the shadow value of the fossil resource increases. The second term shows the total cost of CDR deployment (measured in energy units). According to Proposition 1 optimal deployment monotonically declines over time, and thus optimal emissions decline over time as well.

An increase in $\varphi_{R,0}$ makes the fossil resource a more expensive input for production, and decreases both terms in equation (10). An increase in the carbon dynamics contribution $[(\mathbf{1} - \beta\, \mathbf{\Phi})^{-1}]_{1,i}$ increases the marginal damage from reservoir i. As a result CDR deployment declines, and thus optimal emissions are lower. The outcome of an increase in $[(\mathbf{1} - \beta\, \mathbf{\Phi})^{-1}]_{1,1}$ and ξ_0 is ambiguous as there are two opposing effects. It decreases the first term in equation in (10) but leads to a higher level of CDR which increases the second term.

Using the solutions $G^*_{i,t}$ and E^*_t allows to solve for optimal net energy input I^*_t,

$$I^*_t = \frac{v}{\beta\, \xi_0\, \left[(\mathbf{1} - \beta\, \mathbf{\Phi})^{-1}\right]_{1,1} + (1 - \beta\, \kappa)\beta^{-t} \varphi_{R,0}}. \tag{11}$$

Net energy is defined as the difference between fossil energy and the energy spent on CDR. It is therefore equivalent to the first term in equation (10). Net energy input increases in the energy share v, and decreases in climate change damages ξ_0, the initial resource shadow value $\varphi_{R,0}$, and the carbon dynamics contribution $\left[(\mathbf{1} - \beta\, \mathbf{\Phi})^{-1}\right]_{1,1}$.

In order to gain insides on what changes in the comparison of the global climate economy with and without a CDR technology available, we specify an alternative model by removing the option of CDR from the climate-economy model in section 2. We use the short-hand terminology 'with CDR' for the global climate economy with a CDR technology available and 'without CDR' for the alternative model without the CDR technology; and the short-cut terminology 'CDR does...' when we compare the results from the models with and without CDR. We denote the variables of the alternative model without CDR by a tilde. In the following, we show that the introduction of CDR influences the initial shadow value of the nonrenewable resource, and analyze how this affects net energy input, and net emissions. We discuss the implications of CDR for E^*_t in the subsequent section.

Proposition 3. *CDR increases the shadow value of the fossil resource, and decreases net energy input and net emissions.*

Proof. See Appendix D. □

CDR increases the value of the fossil fuel resource as it creates an additional option to mitigate the negative effects from carbon emissions, and thus reduces the social costs of using fossil fuels. Due to the linear-in-states property of the model, there is no direct effect of CDR on the marginal damage of carbon emissions. As a result, the net effect of CDR on the cost of the fossil resource is positive, and thus net energy input declines, $\Delta\, I_t^* \equiv I_t^* - \tilde{I}_t^* < 0$.

Next, we compare how net emissions differ between both model types. The difference is given by

$$
\begin{aligned}
\Delta E_t^{net^*} &\equiv E_t^{net^*} - \tilde{E}_t^{net^*} \\
&= I_t^* - \tilde{I}_t^* + \sum_{i=2}^{r} \left(f_i(G_{i,t}^*) - G_{i,t}^* \right) < 0,
\end{aligned}
$$

since $\tilde{I}_t^* > I_t^*$ and $f_i(G_{i,t}^*) \leq G_{i,t}^*$.

CDR leads to lower net emission over the entire time path. This result is driven by two effects. First, as already shown CDR lowers net energy input, and second, the cost of CDR is lower than the cost of mitigation (both measured in energy units).

3.3 Social cost of carbon

This section derives the SCC for all reservoir types and explores how CDR influences the first-best carbon tax. Due to the linear-in-states property of the model the marginal damage for each reservoir type is independent of its stock size. This leads to the following result.

Proposition 4. *CDR leaves the structure of the atmospheric SCC (first-best carbon tax) unchanged. The SCC for reservoir i is proportional to net output,*

$$
SCC_{Mi} = Y_t^{net}\, \xi_0 \, \left[(\mathbf{1} - \beta\, \mathbf{\Phi})^{-1} \right]_{1,i} . \tag{12}
$$

As defined above, $[\cdot]_{1,i}$ *denotes the* i^{th} *element of the first column of the inverted matrix in square brackets.*

Proof. See Appendix E. □

The persistence of carbon differs between reservoir types such that each reservoir has its own SCC. For example, for the DICE carbon cycle the carbon dynamics contribution of the deep ocean is smaller than the carbon dynamics contribution of the shallow ocean. This leads to the following ordering: $SCC_{M1} > SCC_{M2} > SCC_{M3}$. The carbon dynamics contribution of the geological reservoirs depend on the rates of leakage to the atmospheric carbon stock. If it is a secure deposit and the leakage rate is zero, then its reservoir specific SCC is zero.

Deriving the atmospheric SCC (first-best carbon tax) for the alternative model specification without CDR leads to the same result as in equation (12). The availability of CDR leaves the analytic structure of the atmospheric SCC unchanged. This result is driven by two crucial assumptions of analytic IAMs. First, utility is a logarithmic function of consumption, and second, climate change damages have an exponential impact on output. Combined with the assumption of full capital depreciation, these two assumptions ensure that the climate-economy model is linear-in-states and can be solved by a linear affine value function (Karp, 2017). The linear-in-states property implies that the marginal damage from an additional unit of carbon in the atmosphere is constant and does not depend on the atmospheric carbon concentration. Hence, removing a unit of carbon from the atmosphere has no effect on the marginal damage, and the atmospheric SCC. This is different for other geoengineering measures such as stratospheric aerosol injections (Meier and Traeger, 2022).

Next, we analyze how the level of the SCC is affected by the availability of CDR, compared to the model without CDR. According to Proposition 3 CDR leads to a lower net energy input over the entire time path. As a result, initial output declines. Since the initial atmospheric carbon concentration and initial climate change damages are equivalent for both model types, initial net output decreases as well. This lowers the initial level of the atmospheric SCC, and therefore rises the level of emissions in the beginning. However, with a CDR technology available, net output must eventually be larger than in the model without CDR technology, provided that CDR is used at all. Thus, there must exist a period in the future in which the SCC is higher compared to the model without CDR. The interpretation of this result is straight forward. As climate change damages are measured in percent of output, an increase in Y_t also increases the money-measured welfare loss from global warming. In other words: The better off the economy is, the more the economy loses from climate change. The next section quantifies this effect.

4 Quantitative analysis

This section illustrates the previous theoretical findings. It provides a calibration of the climate-economy model for a high and low-cost scenario of oceanic CDR, and compares the results to the alternative model specification without CDR.

4.1 Climate-economy model without CDR

The simulation starts in t = 2010 and ends in t = 2200 with one period representing ten years, which is a standard in the literature. Economic growth is driven by increasing total factor productivity $A_{0,t}$, which develops exogenously over time according to

$$A_{0,t} = A_0 \, (1 + w_t)^t, \tag{13}$$

with

$$w_t = w_0 \, (1 + d_w)^{-t}. \tag{14}$$

The initial growth rate of total factor productivity is assumed to be 2 percent per year, $w_0 = 0.02$, and the decline rate $d_w = 0.005$. The initial population is set to 6.9 billion and assumed to grow logistically over time to a maximum of 11 billion in 2200 as in Gerlagh and Lsiki (2018). Output for the initial decade is set to 700 trillion (tn) USD. We use the same shares of capital, $\alpha = 0.3$, and net energy, $v = 0.04$, as in Golosov et al. (2014). The utility discount rate is set to 1.4 percent per year (Traeger, 2022). The given parameter set implies an optimal constant savings rate of $1 - x \approx 0.25$. The initial capital stock is assumed to be 135 trillion USD, approximately the output of two years, and fully depreciates over the course of a decade. We use the carbon cycle from DICE 2013 (Nordhaus and Sztorc, 2013), and the climate change damage parameter $\xi_0 = 5.3 \times 10^{-5}$ from Golosov et al. (2014). The pre-industrial carbon stock is set to 600 GtC. The carbon concentration for the first decade is set to 830.4 GtC yielding initial climate change damages of $D_0 = 1.2$ percent.

Assuming emissions of 86.7 GtC for the first decade (Gerlagh and Lsiki, 2018) allows to solve for the initial level of total factor productivity, and delivers $A_0 = 38$. We then calibrate the initial resource stock such that it matches the initial level of emissions. This implies an initial fossil fuel stock size of 793.25 GtC. Table 1 summarizes the model parameters and initial stock values.

Table 1: Parameter values

K_0	N_0	R_0	κ	v	β	w	A_0	ξ_0
135	6.9	793.25	0.3	0.04	0.986	0.02	38.02	5.3×10^{-5}
trillion USD	billion	GtC			1/year	1/year		1/GtC

Figure 1 shows the outcome of the global climate-economy model without CDR technology available. The fossil resource is scarce and almost entirely used up over the time horizon considered. Emissions start at 86 GtC per decade and monotonically decline over time, as the shadow price of the resource increases. Damages start at 1.2 percent of global output and increase up to around 3 percent by the year 2100. Afterwards, damages start to decline as less energy is used and more carbon is taken up by the ocean. Relative net production (GDP) rises over time due to the growth of total factor productivity. The atmospheric SCC starts at around 45 USD/tCO_2 and increases up to around 800 USD/tCO_2 by the year 2100. All these results are very much in line with results of common IAMs (e.g. Golosov et al., 2014).

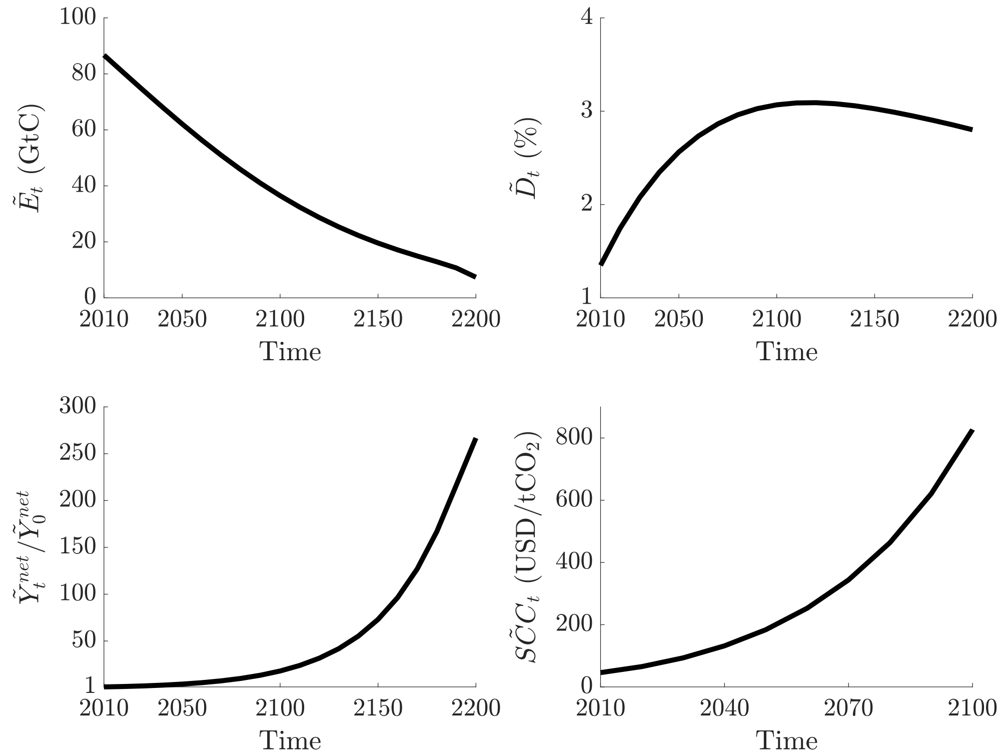

Figure 1: The graph shows emissions per decade ($\tilde{E}_t$), damages ($\tilde{D}_t$), relative net output ($\tilde{Y}_t^{net}/\tilde{Y}_0^{net}$), and the social cost of carbon ($S\tilde{C}C_t$) for the calibrated standard climate-economy model without CDR.

4.2 Climate-economy model with marine CDR

This section introduces the option of marine CDR and explores how it affects the outcome of the standard climate-economy model. Cost estimates for the storage of carbon in the ocean are still uncertain and vary widely. IPCC (2005) estimates the cost for oceanic storage between 22 and 114 USD/tC. Rickels et al. (2018) consider a convex cost function with a broad parameter range for the quadratic cost term to account for uncertainty about the cost of large-scale deployment.

To capture the cost uncertainty for marine CDR we consider a low and high-cost scenario. For the low-cost case, the cost function for CDR is given by

$$f_l(G_t) = g_l\, G_t^2, \tag{15}$$

with parameter g_l to be calibrated. As a point of reference, we use the linear quadratic cost function from Rickels et al. (2018) and combine it with the lower bound cost estimate for oceanic storage of 22 USD/tC from IPCC (2005), which leads to

$$F(G_t) = 0.022\, G_t + 0.01833\, G_t^2. \tag{16}$$

CDR deployment G_t is measured in GtC and $F(G_t)$ shows the costs in trillion USD (tn USD). We calibrate the cost function $f_l(G_t)$ to equation (16) for the initial time period. Minimizing the squared difference over the interval $G_t \in (0, 18.5)$ yields $g_l = 0.056$. We choose this interval since for $G_t \geq 18.5$ the cost of CDR is higher than the cost of mitigation. Figure 2 shows the quality of the fit, and the cost of mitigation in trillion USD. For the high-cost scenario, we consider the upper bound

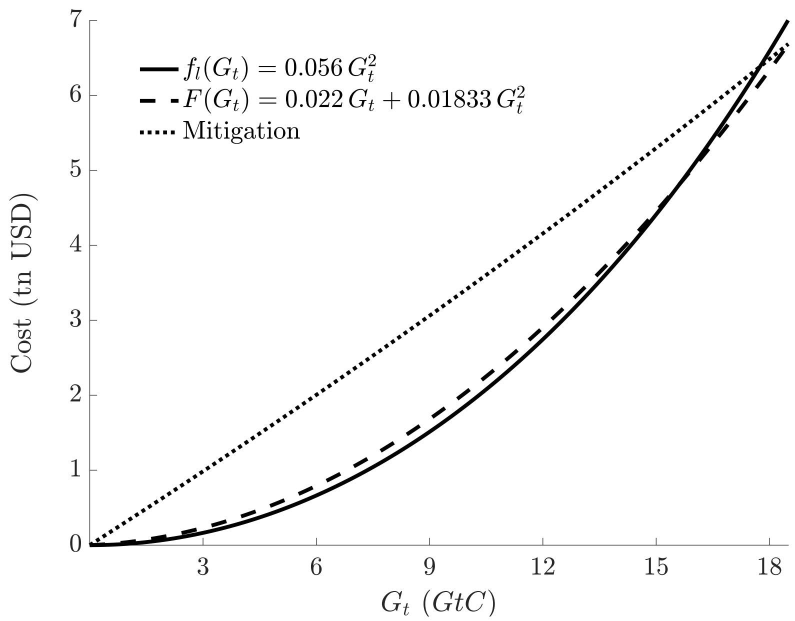

Figure 2: The graph shows the calibrated cost function $f_l(G_t)$ (solid line), the cost estimate based on Rickels et al. (2018) and IPCC (2005) (dashed line), and the cost of mitigation (dotted line).

of previous estimates. As the upper bound cost estimate is expected to surpass the lower bound cost estimate by a factor of five (IPCC, 2005), we assume $g_h = 5 \times g_l$. Due to the assumption of a quadratic cost function the level of CDR will still be positive but considerably lower than in the low-cost scenario. Figures 3 and 4 show how the results change due to the introduction of CDR. The black solid lines show the results for the low-cost scenario and the dotted green lines show the outcome for the high-cost scenario.

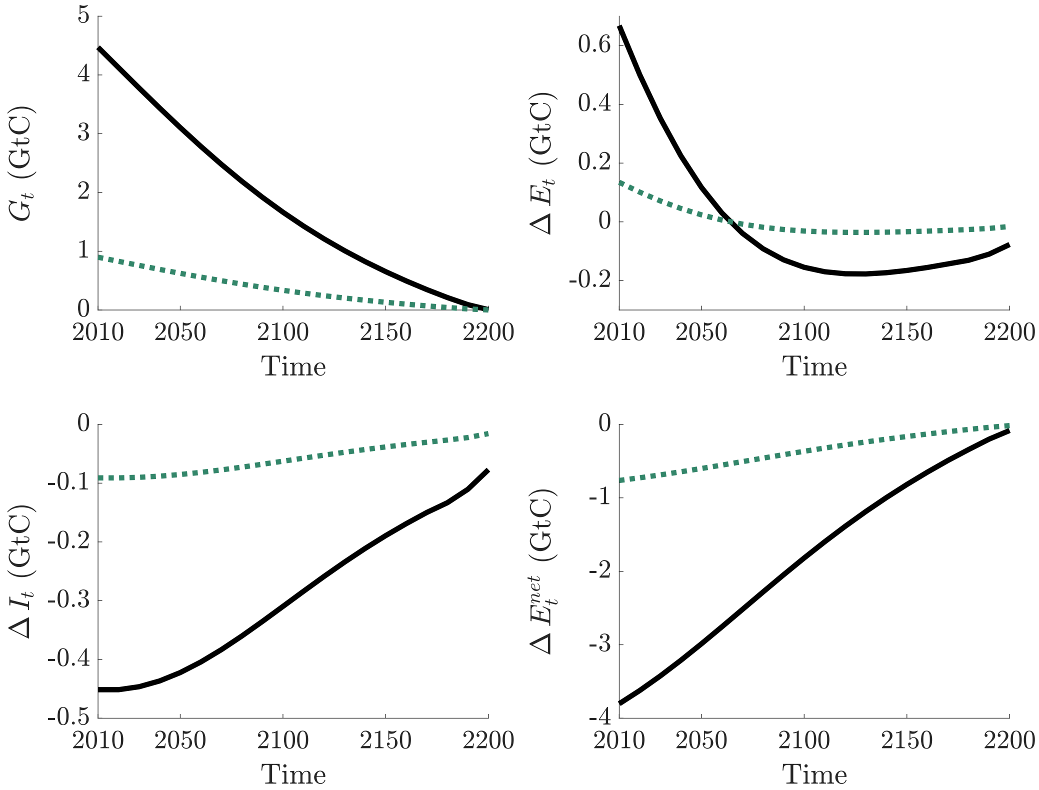

Figure 3: The graph shows the optimal deployment of oceanic CDR (G_t) per decade and the difference in emissions ($\Delta\, E_t$), net energy input ($\Delta\, I_t$), and net emissions ($\Delta\, E_t^{net}$) compared to the outcome of the standard model without CDR for the low cost (black solid lines), and high cost scenario (green dotted lines).

The simulation illustrates the analytic results from the previous section. In the first decade, in the low-cost case around 4.5 GtC are removed from the atmosphere and stored in the deep ocean. In the high-cost scenario, CDR deployment is considerably lower with only 1 GtC in the first decade. As described in Propositions

1 and 2 CDR deployment and emissions monotonically decline over time. In line with Proposition 3, net emissions and net energy input is lower over the entire time horizon compared to the model without CDR. In both scenarios emissions are first higher and then lower than in the model without CDR. In the low cost scenario the difference is more pronounced.

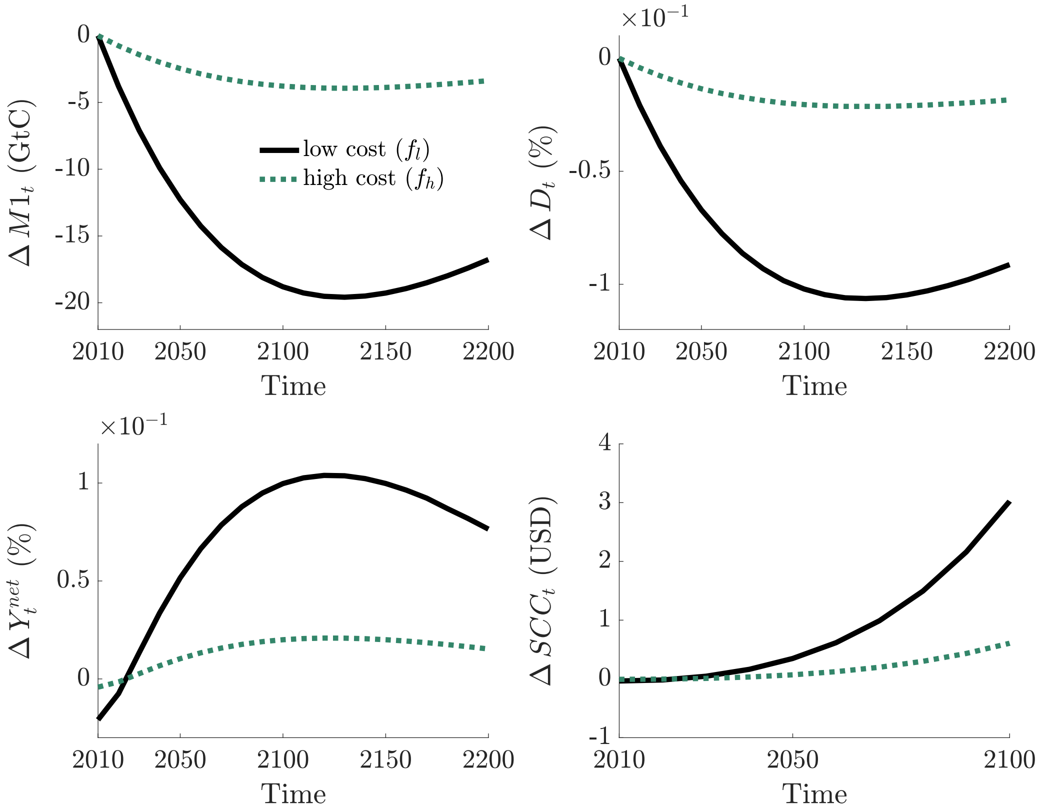

Figure 4: The graph shows the difference in atmospheric carbon concentration ($\Delta\, M_{1,t}$), climate change damages ($\Delta\, D_t$), net output ($\Delta\, Y_t^{net}$), and the social cost of carbon ($\Delta\, SCC_t$) compared to the outcome of the standard model without CDR for the low cost (black solid lines), and high cost scenario (green dotted lines).

In the low-cost case, CDR reduces the atmospheric carbon concentration by 20 GtC in 2125 and damages are lower by around 0.1 percentage points of output. Towards the end, the negative effect on the atmospheric carbon concentration and climate damages wears off as CDR deployment goes to zero and more and more carbon has cycled back from the oceans. In the high-cost case, the negative effect on atmospheric carbon is minor and only decreases damages by around 0.01 percent.

The numerical simulation also allows to assess how strongly net output and the atmospheric SCC (optimal carbon tax) are affected by the introduction of CDR. As already discussed in the theoretical part of the paper initial net output declines as CDR becomes available. Figure 4 shows that this effect is rather small. Net output declines by 0.025 percent in the low-cost scenario. Afterwards, the effect on net output becomes positive and grows until 2125 to around 0.11 percent. Similar to net output, the SCC is first lower and then higher. The economy first emits more and then less. The simulation shows that the effect of CDR on the SCC is minor. By 2100 the SCC is only higher by 3 USD/tCO_2 compared to the model without CDR.

4.3 Ocean CDR in horizontally and vertically differentiated carbon cycles

The analysis of ocean CDR in the previous section considers the ocean CDR decision from a global perspective where the cost function summarizes the different ocean CDR options. However, countries face different ocean CDR options (reflecting their specific conditions) which in turn translates into different CDR cost functions. For example, a country might have good prospects for blue carbon projects (due to mangrove coastal ecosystems) which provides it with a very low, initial cost but limited overall potential. Accordingly, this country might face a very steep cost function. Another country might face conditions suited for ocean alkalinization which starts with a relatively high initial cost compared to blue carbon but the increase in marginal cost is rather flat if extending the scale. Furthermore, even for the same ocean CDR technology, countries might face different cost due to the availability of energy input or minerals. Finally, countries might follow different strategies regarding ocean CDR (e.g. due to variation in public acceptance which also implies different (social) costs).

Accordingly, investigation of ocean CDR requires considering regional deployment scenarios. The general structure of the carbon cycle in (3) allows for such an investigation. Figure 5 shows ocean CDR deployment under an equal split of the upper ocean where two countries, A and B, decide separately about ocean CDR in their access to the ocean where both countries face the same cost function. Here, the cost function reflect that the scarcities determining the convexity of the cost function are assumed to be locally and hence each country deploys ocean CDR in its regional basin, resulting in a slightly lower atmospheric carbon concentration. Compared to the previous section, the carbon cycle transition matrix (3) is now a $4x$ matrix to capture the additional fluxes due to the split of the upper ocean.

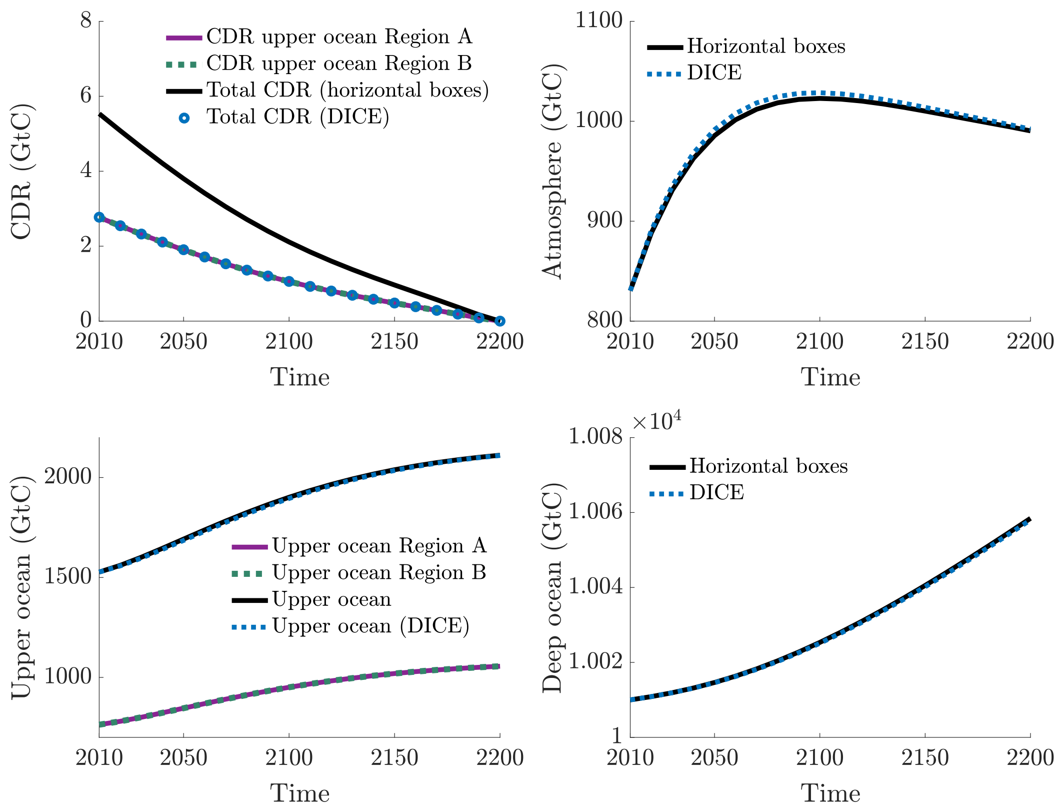

Figure 5: Optimal CDR with a horizontally differentiated carbon cycle. The graph assumes identical CDR cost functions for Region A and B.

The Figure 6 assumes the same carbon cycle, but with differentiated costs. Accordingly, the CDR deployment is regionally unevenly distributed. The carbon cycle transition matrix (3) can be augmented such that N regions are included which induce ocean CDR in their "coastal" upper ocean, facing different ocean CDR cost and different CO_2-air-sea-flux conditions, making them differently suited for specific ocean CDR methods.

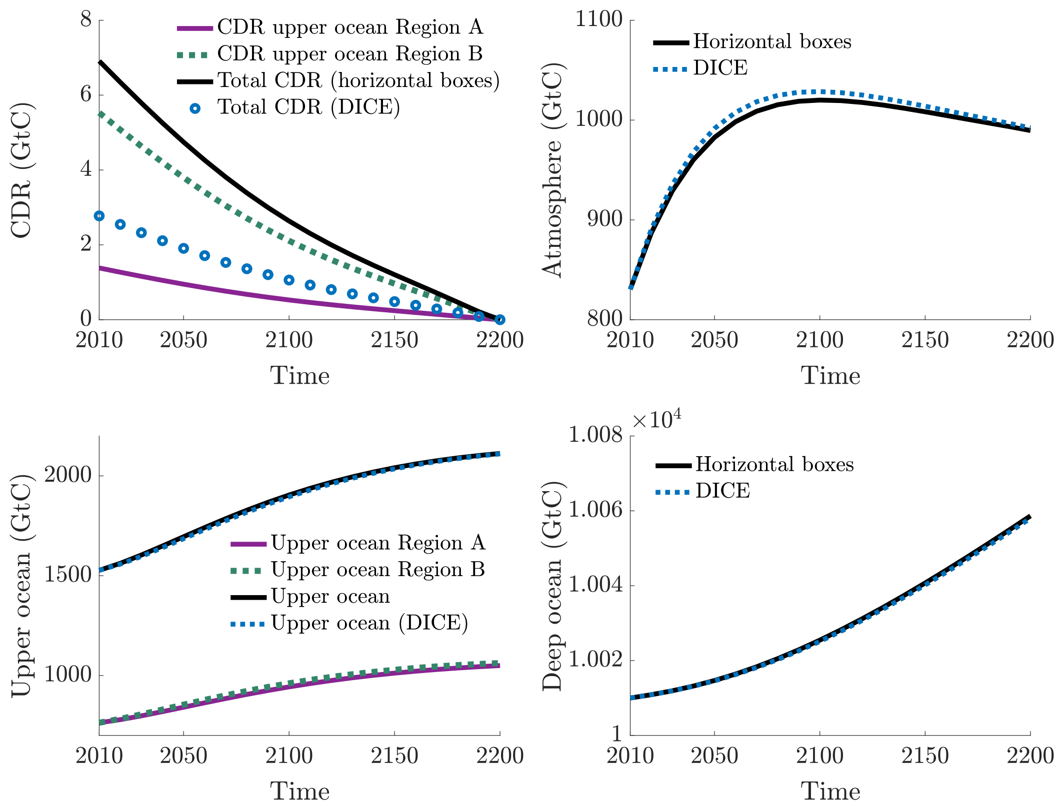

Figure 6: Regionalized CDR cost functions. We increase the cost function for Region A by a factor of 2 and decrease the cost function for Region B by a factor of 0.5.

The ocean CDR methods differ also by the depth of carbon storage. Furthermore, for various ocean CDR methods like for example direct CO_2 injection or biomass dumping, the storage depth influences the CDR cost. In Figure 7 we show the variation in storage depth for equal cost, indicating the benefits of deeper storage. Again, we augment (3) to be a $4x4$ matrix, now including a more detailed representation of the mixed layer and deep ocean. We distinguish between an upper and lower mixed layer (i.e. ocean) and a deep ocean. The figure shows the optimal CDR deployment scenario for each storage depth, confirming that ocean

CDR which stores carbon below the upper mixed layer is more beneficial that upper ocean methods. However, once the storage has been achieved a depth below the upper layer, there is only small additional benefit from storing carbon in the deep ocean compared to the lower ocean.

Figure 7 shows the results for the vertical split of the deep ocean.

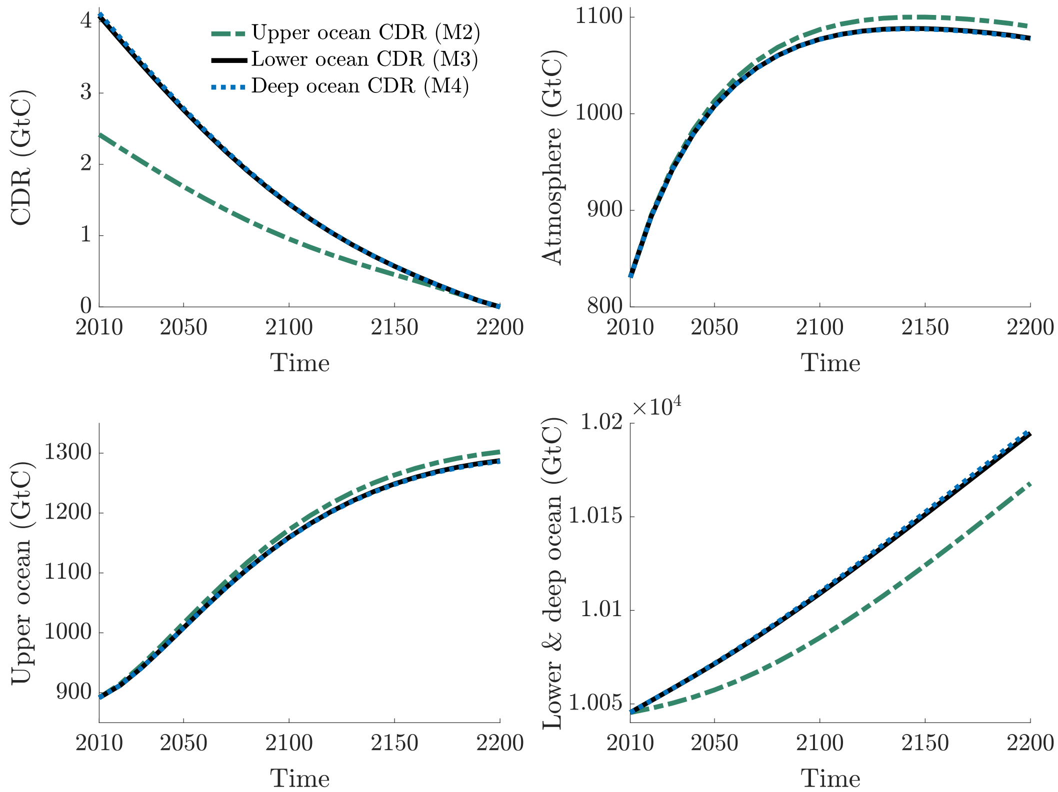

Figure 7: Optimal CDR with a vertically differentiated carbon cycle. The graph assumes identical CDR cost functions for lower and deep ocean.

5 Summary and conclusions

The paper introduces the option of carbon dioxide removal (CDR) and storage in different reservoir types into an analytic climate-economy model and compares the results to a model variant without CDR. The analytic model shows that the availability of CDR alters the level of the SCC. However, the quantitative analysis suggests that this effect is negligible. In the low-cost scenario, CDR increases initial emissions by around 0.6 GtC, which is equivalent to around 0.7 percent of total carbon emissions. Thus, with an optimal policy in place the introduction of CDR has hardly any effect on mitigation incentives. The model suggests that it is optimal to use CDR on top of traditional mitigation efforts.

Furthermore, the present analysis provides basic implications for the optimal implementation of CDR technologies. One option that has been proposed in the literature is the introduction of a differentiated carbon tax (Rickels and Lontzek, 2012). This paper presents a simple formula for the reservoir-specific carbon tax, and characterizes its components. Another suggestion for the optimal implementation of CDR is the introduction of carbon credits (Chomitz and Lecocq, 2004; Sedjo and Marland, 2003), for which this paper also offers a simple way to calculate it. The analytical structure with the different social cost for the various boxes allows assessing a broad variety of marine CDR options by considering different boxes in the carbon cycle. This provides the basis for the analysis of decentralized and potentially non-cooperative CDR policies.

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Appendices

A Solving the linear-in-states model

For the proof of the linear-in-states property I follow Traeger (2022). The consumption rate can be written as

$$x_t = \frac{C_t}{Y_t\,\left[1 - D_t\,(M_{1,t})\right]},$$

such that

$$\log C_t = \log x_t + \log A_{0,t} + \kappa \log K_t + (1 - \kappa - v) \log N_{0,t} + v \log I_t - \xi_0\,(M_{1,t} - M_1^{pre}).$$

I transform the optimization problem into its dynamic programming form (Bellman equation)

$$V(k_t, \boldsymbol{M}_t, R_t, t) = \max_{x_t, E_t, G_t} \Bigg\{ \log x_t + \log A_{0,t} + \kappa \log K_t + (1 - \kappa - v) \log N_{0,t}$$
$$+ v \log I_t(E_t, G_{i,t}) - \xi_0\,(M_{1,t} - M_1^{pre}) + \beta\, V(k_{t+1}, \boldsymbol{M}_{t+1}, R_{t+1}, t+1) \Bigg\},$$

where $k_t = \log K_t$ with the equation of motion

$$k_{t+1} = \log A_{0,t} + \kappa \log K_t + (1 - \kappa - v) \log N_{0,t} + v \log I_t - \xi_0\,(M_{1,t} - M_1^{pre}) + \log(1 - x_t). \tag{17}$$

To solve the intertemporal optimization problem, I use the following guess for the value function

$$V(k_t, \boldsymbol{M}_t, R_t, t) = \varphi_k\, k_t + \boldsymbol{\varphi}_M^T\, \boldsymbol{M}_t + \varphi_{R,t}\, R_t + \varphi_t, \tag{18}$$

where φ is used to denote the shadow values for the different states, and ^{T} denotes the transpose of a vector of shadow values.

Inserting the trial solution and the next periods states (equations 2, 3, and 17) into the Bellman equation delivers

$$\begin{aligned}
&\varphi_k\, k_t + \boldsymbol{\varphi}_M^T\, \boldsymbol{M}_t + \varphi_{R,t}\, R_t + \varphi_t \\
&= \max_{x_t, E_t, G_{i,t}} \Bigg\{ \log x_t + \log A_{0,t} + \kappa\, k_t + (1-\kappa-v) \log N_{0,t} + v \log I_t(E_t, G_{i,t}) - \xi_0\, (M_{1,t} - M_1^{pre}) \\
&+\beta\, \varphi_k \big(\log A_{0,t} + \kappa\, k_t + (1-\kappa-v) \log N_{0,t} + v \log I_t(E_t, G_{i,t}) - \xi_0\, (M_{1,t} - M_1^{pre}) + \log(1-x_t) \big) \\
&\qquad + \beta\, \boldsymbol{\varphi}_M^T\, (\boldsymbol{\Phi} \boldsymbol{M}_t + \boldsymbol{E}_t) + \beta\, \varphi_{R,t+1}\, (R_t - E_t) + \beta\, \varphi_{t+1} \Bigg\}. \qquad (19)
\end{aligned}$$

First order conditions. Maximizing the right hand side over x_t yields

$$\frac{1}{x_t} - \beta\, \varphi_k \frac{1}{1-x_t} = 0 \quad \Longrightarrow \quad x_t^* = \frac{1}{1+\beta\, \varphi_k}. \qquad (20)$$

Next, I find the first order condition for CDR deployment for reservoir i

$$-v(1+\beta\, \varphi_k)\, \frac{f_i'(G_{i,t})}{I_t} = \beta(\varphi_{M1} - \varphi_{Mi}), \qquad (21)$$

and the first order condition for emissions

$$v(1+\beta\, \varphi_k)\, \frac{1}{I_t} = \beta(\varphi_{R,t+1} - \varphi_{M1}). \qquad (22)$$

Inserting (22) into (21) and solving for $G_{i,t}$ leads to

$$G_{i,t}^* = f_i'^{-1} \left(\frac{\varphi_{M1} - \varphi_{Mi}}{\varphi_{M1} - \varphi_{R,t+1}} \right), \qquad (23)$$

where the inverse of the marginal cost function is denoted by $f_i'^{-1}$. Summing up

CDR deployment over all reservoir types yields

$$G_{1,t}^* = \sum_{i=2}^{r=4} f_i'^{-1}\left(\frac{\varphi_{M1} - \varphi_{Mi}}{\varphi_{M1} - \varphi_{R,t+1}}\right)$$

Using (23) and solving for optimal emissions yields

$$E_t^* = \frac{v(1+\beta\,\varphi_k)}{\beta(\varphi_{R,t+1} - \varphi_{M1})} + \sum_{i=2}^{r=4} f_i\left(f_i'^{-1}\left(\frac{\varphi_{M1} - \varphi_{Mi}}{\varphi_{M1} - \varphi_{R,t+1}}\right)\right). \tag{24}$$

First order conditions deliver optimal controls x_t^*, E_t^*, and $G_{i,t}^*$ which are independent of the states.

Using E_t^* and $G_{i,t}^*$ one can solve for the optimal net energy input I_t^*.

$$I_t^* = E_t^* - \sum_{i=2}^{r=4} f_i(G_{i,t}^*) = \frac{v(1+\beta\,\varphi_k)}{\beta(\varphi_{R,t+1} - \varphi_{M1})}. \tag{25}$$

Inserting the optimal controls into (19) and arranging terms with respect to their states yields

$$\begin{aligned}\varphi_k\,k_t + \boldsymbol{\varphi}_M^T\,\boldsymbol{M}_t + \varphi_{R,t}\,R_t + \varphi_t = \Big[(1+\beta\,\varphi_k)\kappa\Big]k_t + \Big[\beta\,\boldsymbol{\Phi}\boldsymbol{\varphi}_M^T - (1+\beta\,\varphi_k)\xi_0\,\boldsymbol{e}_1^T\Big]\boldsymbol{M}_t \\ +\Big[\beta\,\varphi_{R,t+1}\Big]R_t + \log\,x_t^* + \beta\,\varphi_k\,\log(1-x_t^*) + (1+\beta\,\varphi_k)\log A_{0,t} + (1+\beta\,\varphi_k)(1-\kappa-v)\log N_{0,t} \\ +(1+\beta\,\varphi_k)v\log I_t^* + (1+\beta\,\varphi_k)\xi_0\,M_1^{pre} + \beta\,\varphi_{M1}(E_t^* + E_t^{\text{exo}} - G_{1,t}^*) + \beta\,\varphi_{M2}\,G_{2,t}^* + \beta\,\varphi_{M3}\,G_{3,t}^* \\ + \beta\,\varphi_{M4}\,G_{4,t}^* - \beta\,\varphi_{R,t+1}\,E_t^* + \beta\,\varphi_{t+1}.\end{aligned} \tag{26}$$

Given the optimal controls the maximized Bellman equation is linear in all states.

Shadow values. Coefficient matching with respect to capital, k_t, yields

$$\varphi_k = (1+\beta\,\varphi_k)\kappa \quad \Leftrightarrow \quad \varphi_k = \frac{\kappa}{1-\beta\,\kappa} \tag{27}$$

Inserting φ_k into equation (20) yield the optimal consumption rate $x_t^* = 1 - \beta\,\kappa$.

I match the coefficients of each state from both sides of the equation, which leads to

$$\boldsymbol{\varphi}_M^T = -\xi_0 \, (1 + \beta \, \varphi_k) \, \boldsymbol{e}_1^T \, [\mathbf{1} - \beta \, \boldsymbol{\Phi}]^{-1}$$

Using (27) the vector of shadow prices turns to

$$\boldsymbol{\varphi}_M^T = -\xi_0 \, \frac{1}{1 - \beta \, \kappa} \, \boldsymbol{e}_1^T \, [\mathbf{1} - \beta \, \boldsymbol{\Phi}]^{-1} \tag{28}$$

Coefficient matching with respect to the resource stock yields

$$\varphi_{R,t} = \beta \, \varphi_{R,t+1} \quad \Leftrightarrow \quad \varphi_{R,t} = \beta^{-t} \varphi_{R,0} \quad \text{(Hotelling's rule).} \tag{29}$$

The initial resource values $\varphi_{R,0}$ depend on the set up of the economy, including assumptions about production and the energy sector. Given the coefficients and the optimal rate of consumption equation (26) turns to the following condition:

$$\begin{aligned} \varphi_t - \beta \, \varphi_{t+1} &= \log \, x_t^* + \beta \, \varphi_k \, \log(1 - x_t^*) + (1 + \beta \, \varphi_k) \log A_{0,t} + (1 + \beta \, \varphi_k)(1 - \kappa - v) \log N_{0,t} \\ &+ (1 + \beta \, \varphi_k) v \log I_t^* + (1 + \beta \, \varphi_k) \xi_0 \, M_1^{pre} + \beta \, \boldsymbol{\varphi}_M^T \, \boldsymbol{E}_t^* - \beta \, \varphi_{R,t+1} \, E_t^* \end{aligned}$$

This condition will be satisfied by picking the sequence $\varphi_0, \varphi_1, \varphi_2, \ldots$. The additional condition $\lim_{t \to \infty} \beta^t V(\cdot) = 0 \Rightarrow \lim_{t \to \infty} \beta^t \varphi_t = 0$ pins down this initial value φ_0.

B Proof of Proposition 1

Inserting the solutions for the shadow values, equations (27) to (29), into (23) yields

$$G_{i,t}^* = f_i'^{-1} \left(\frac{\beta \, \xi_0 \, [(\mathbf{1} - \beta \, \boldsymbol{\Phi})^{-1}]_{1,1} - \beta \, \xi_0 \, [(\mathbf{1} - \beta \, \boldsymbol{\Phi})^{-1}]_{1,i}}{\beta \, \xi_0 \, [(\mathbf{1} - \beta \, \boldsymbol{\Phi})^{-1}]_{1,1} + (1 - \beta \, \kappa) \beta^{-t} \varphi_{R,0}} \right), \tag{30}$$

where $[\cdot]_{1,1}$ denotes the first, and $[\cdot]_{1,i}$ denotes the i^{th} element of the first column of the inverted matrix in square brackets. Note that $\left[(\mathbf{1}-\beta\,\boldsymbol{\Phi})^{-1}\right]_{1,1} > \left[(\mathbf{1}-\beta\,\boldsymbol{\Phi})^{-1}\right]_{1,i}$.

C Proof of Proposition 2

Inserting the solutions for the shadow values, equations (27) to (29), into (24) yields

$$E_t^* = \frac{v}{\beta\,\xi_0\,\left[(\mathbf{1}-\beta\,\boldsymbol{\Phi})^{-1}\right]_{1,1} + (1-\beta\,\kappa)\beta^{-t}\varphi_{R,0}} + \sum_{i=2}^{r=4} f_i\left(G_{i,t}^*\right), \tag{31}$$

where

$$G_{i,t}^* = f_i'^{-1}\left(\frac{\beta\,\xi_0\left[(\mathbf{1}-\beta\,\boldsymbol{\Phi})^{-1}\right]_{1,1} - \beta\,\xi_0\left[(\mathbf{1}-\beta\,\boldsymbol{\Phi})^{-1}\right]_{1,i}}{\beta\,\xi_0\,\left[(\mathbf{1}-\beta\,\boldsymbol{\Phi})^{-1}\right]_{1,1} + (1-\beta\,\kappa)\beta^{-t}\varphi_{R,0}}\right).$$

D Proof of Proposition 3

Consider the climate-economy model from section 2 without the option of CDR, and let the variables of this model specification be denoted by a tilde.

From the first order condition (21) it follows that optimal emissions without the option of CDR are given by

$$\tilde{E}_t^* = \frac{v}{\beta\,\xi_0\,\left[(\mathbf{1}-\beta\,\boldsymbol{\Phi})^{-1}\right]_{1,1} + (1-\beta\,\kappa)\beta^{-t}\tilde{\varphi}_{R,0}}. \tag{32}$$

The only endogenous term in equation (32) is the initial shadow value of the resource stock, which is denoted by $\tilde{\varphi}_{R,0}$. In both model specifications, the size of the resource stock is the same and will be used up eventually. Therefore,

$$R_0 = \sum_{t=0}^{\infty} E_t^* = \sum_{t=0}^{\infty} \tilde{E}_t^*.$$

Using equations (31) and (32), and rearranging leads to

$$\sum_{t=0}^{\infty}\sum_{i=2}^{r=4} f_i(G^*_{i,t}) = \sum_{t=0}^{\infty}\Bigg(\frac{v}{\beta\,\xi_0\,\left[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}\right]_{1,1} + (1-\beta\,\kappa)\beta^{-t}\tilde{\varphi}_{R,0}} - \frac{v}{\beta\,\xi_0\,\left[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}\right]_{1,1} + (1-\beta\,\kappa)\beta^{-t}\varphi_{R,0}}\Bigg).$$

If there exists at least one point in time where $\sum_{i=2}^{r=4} f_i(G^*_{i,t}) > 0$, the left term of the equation is positive, and thus $\tilde{\varphi}_{R,0} < \varphi_{R,0}$. From this it directly follows that $\Delta\, I^*_t \equiv I^*_t - \tilde{I}^*_t < 0$.

Comparing net emissions with and without the option of CDR yields

$$\begin{aligned}\Delta E^{net^*}_t &\equiv E^{net^*}_t - \tilde{E}^{net^*}_t \\ &= I^*_t - \tilde{I}^*_t + \sum_{i=2}^{r=4}\left(f_i(G^*_{i,t}) - G^*_{i,t}\right) < 0,\end{aligned}$$

since $\tilde{I}^*_t > I^*_t$ and $f_i(G_{i,t}) \leq G^*_{i,t}$.

E Proof of Proposition 4

The SCC is the negative of the shadow value of carbon reservoir i expressed in money-measured consumption units,

$$\begin{aligned} SCC_{Mi} &= -(1-\beta\,\kappa)Y^{net}_t\,\varphi_{Mi} \\ &= Y^{net}_t\,\xi_0\,\left[(\mathbf{1}-\beta\,\mathbf{\Phi})^{-1}\right]_{1,i},\end{aligned}$$

where again $[\cdot]_{1,i}$ denotes the i^{th} element of the first column of the inverted matrix in square brackets.